UNIT



Patterns and Relations

Just for Fun

Word Find

Find the following words in the puzzle below. You can move in any direction. term divisibility rule unit tile

d	i	V	i	S
е	t	е	Ι	i
	m	r	i	b
u	n	i	t	i
r	У	t	i	I

Pattern Search

Choose a grid of any 4 squares in the calendar. What patterns do you see in the numbers?

			May			
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

Variation: Choose a grid of any 9 squares or pick a different month and try again.

A Game for **2** or more

Express Yourself!

Make as many words as you can from the letters of the words "algebraic expression." The person with the most words after 3 minutes wins!

Activating Prior Knowledge

Order of Operations

Perform operations inside the brackets first. Next, divide and multiply in order from left to right. Then add and subtract in order from left to right. The letters B, D, M, A, and S can help you remember the order of operations. B—Brackets D, M—Divide, Multiply A, S—Add, Subtract

Example 1

Simplify.

a) 10 − 3 × 2

b) $12 \div (5+1)$ **c)** $6 \times 2 \div 3 + 1$

Solution

a) $10 - 3 \times 2 = 10 - 6$	Multiply first.
= 4	Then subtract.
b) $12 \div (5+1) = 12 \div 6$	Add inside the brackets first.
= 2	Then divide.
c) $6 \times 2 \div 3 + 1 = \underbrace{12 \div 3}_{= 4 + 1} + 1 = 5$	Multiply first. Then divide. Then add.



1. Simplify.

a) 12 - 2 × 4	b) 20 ÷ (2 + 3)	c) $12 \div 6 \times 5 + 4$
=12	= 20 ÷	= × 5 + 4
=	=	=
d) 10 + 4 ÷ 2	e) (9 − 5) × 6	f) $5 + 2 \times 3 - 4$
= 10 +	= × 6	= 5 +
=	=	=

Graphing on a Coordinate Grid

An ordered pair, such as (5, 3), tells you the position of a point on a grid. The first number is the horizontal distance from the origin. The second number is the vertical distance from the origin.

The numbers of an ordered pair are also called the coordinates of a point.

Example 2

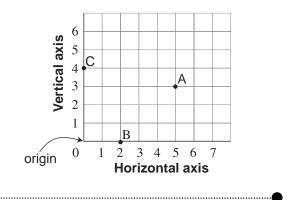
Graph the points A(5, 3), B(2, 0), and C(0, 4) on a grid.

Solution

To plot point A, start at 5 on the horizontal axis, then move up 3.

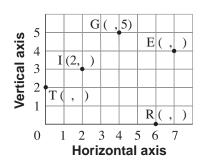
To plot point B, start at 2 on the horizontal axis, then move up 0. Point B is on the horizontal axis.

To plot point C, start at 0 on the horizontal axis, then move up 4. Point C is on the vertical axis.





2. Write the ordered pair for each point on the grid.

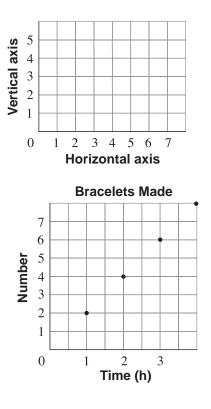


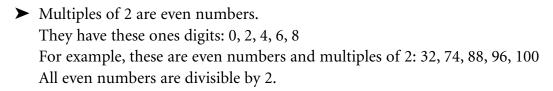
- **4.** The graph shows the number of bracelets Jan can make over time.
 - a) How many bracelets can Jan make

in 3 h? _____

- **b)** How long will it take to make
 - 10 bracelets? _____

3. Plot and label these points: A(0, 5), B(2, 4), E(4, 3), R(5, 0)





A number is a multiple of 4 if the tens and ones digits of the number form a number that is a multiple of 4.
 For example, 124 is a multiple of 4 because 24 is a multiple of 4.
 And, 3036 is a multiple of 4 because 36 is a multiple of 4.

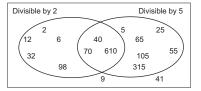
Multiples of 5 have these ones digits: 0, 5
 For example, these numbers are multiples of 5: 5, 20, 45, 350
 Multiples of 5 are divisible by 5.

A number is a multiple of 8 if the hundreds, tens, and ones digits of the number form a number that is a multiple of 8.
 For example, 1888 is a multiple of 8 because 888 is a multiple of 8.
 And, 1040 is a multiple of 8 because 040, or 40 is a multiple of 8.

 Multiples of 10 have a ones digit that is 0.
 For example, these numbers are multiples of 10: 20, 40, 130, 770 Multiples of 10 are divisible by 10.

You can use a Venn diagram to show numbers that are divisible by two or more numbers.

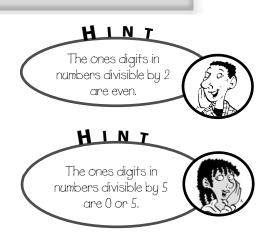
This Venn diagram shows divisibility by 2 and by 5.

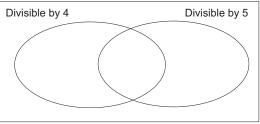


Multiples of 2 are in the left loop. Multiples of both 2 and 5 are in the middle loop. Multiples of 5 are in the right loop. Numbers that are *not* multiples of 2 or of 5 are outside the loops.

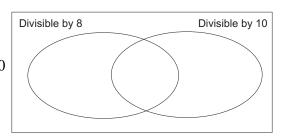


- **1.** Circle the numbers that are divisible by 2.
 - 23 98 21 44 11 77 34
- **2.** Circle the numbers that are divisible by 5.
 - 55 10 7 59 105 775 1025
- 3. Circle the numbers that are divisible by 2 and by 5.
 10 30 25 55 1000 52
- Write each number in the correct place in the Venn diagram.
 16, 20, 33, 64, 80, 95, 97, 105, 214, 216, 324, 405





- **5.** Write four 3-digit numbers that are divisible by 10.
- **6.** Write three 4-digit numbers that are divisible by 8.
- 7. a) Write each number in the correct place in the Venn diagram.
 115, 116, 120, 168, 450, 753, 800, 928, 1008, 1110



b) Write 4 more numbers in the Venn diagram – one in each loop and one outside the loops. How do you know you placed each number correctly?



➤ A number is divisible by 3 if the sum of its digits is divisible by 3. For example, 1035 is divisible by 3 because 1 + 0 + 3 + 5 = 9, and 9 is divisible by 3. 1036 is *not* divisible by 3 because 1 + 0 + 3 + 6 = 10, and 10 is *not* divisible by 3.

A number is divisible by 6 if the number is divisible by 2 and by 3. For example, 1038 is divisible by 2 because the number is even.
 1038 is divisible by 3 because 1 + 0 + 3 + 8 = 12, which is divisible by 3.
 So, 1038 is divisible by 6.

➤ A number is divisible by 9 if the sum of its digits is divisible by 9. For example, 5418 is divisible by 9 because 5 + 4 + 1 + 8 = 18, and 18 is divisible by 9. 5428 is *not* divisible by 9 because 5 + 4 + 2 + 8 = 19, and 19 is *not* divisible by 9.

► No number is divisible by 0.

You can use a Carroll diagram to show numbers that are divisible by two numbers. This Carroll diagram shows divisibility by 6 and by 9.

	Divisible by 6	Not divisible by 6
Divisible by 9	18, 36, 126, 162	27, 45, 963, 711
Not divisible by 9	6, 12, 204, 402	10, 29, 325, 802

➤ You can use divisibility rules to help list the factors of a number. To list the factors of 156: Try each rule in turn. Divide by 2: 156 ÷ 2 = 78 Divide by 3: 156 ÷ 3 = 52 Divide by 4: 156 ÷ 4 = 39
156 is not divisible by 5. Divide by 6: 156 ÷ 6 = 26
156 is not divisible by 7, by 8, by 9, or by 10. Use a calculator to check for divisibility by 11 and 12.
156 is not divisible by 11. Divide by 12: 156 ÷ 12 = 13
Since the factors 12 and 13 are close in value, you have found all the factors. In order, the factors of 156 are: 1, 2, 3, 4, 6, 12, 13, 26, 39, 52, 78, 156

1. Match the number with the correct divisibility statement. Draw more than one line if it is needed.

54	Divisible by 10.
56	Divisible by 3.
50	Divisible by 9.
92	Divisible by 8.
75	Divisible by 5.
93	Divisible by 2.
30	Divisible by 6.

2. Cross out the numbers that are *not* divisible by 2.

12 79 98 134 227 2	2469
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How do you know the numbers are not divisible by 2?

3. Circle the numbers that are divisible by 9.

91 331 333 153 99 12 321

How do you know you are correct?

4. Write four numbers that are divisible by 6: ______ How did you choose those numbers?

5. Solve each riddle.

a) I am divisible by 2 and by 3.I am between 21 and 29.Which number am I? _____

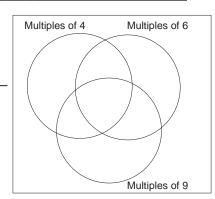
- b) I am divisible by 5 and by 10.I am between 56 and 64.Which number am I? _____
- c) I am divisible by 2 and by 9. I am between 424 and 449. Which number am I?

6. Which numbers below are divisible by 3? By 6? By 9? How do you know?

	a)	124
		215
	ς,	330
	d)	450
	e)	150
	-,	
7.	Us	e your answers to question 6 to help you list the factors of each number.
	a)	124:
	b)	215:
	c)	150:
8.		Sort these numbers in the Carroll diagram below.
	-	16, 18, 27, 37, 120, 180, 281, 288, 352, 411, 432, 540

	Divisible by 9	Not divisible by 9
Divisible by 4		
Not divisible by 4		

- **b)** Write one more number in each part of the Carroll diagram. Explain how you knew where to place each number.
- **9.** a) Sort these numbers in the Venn diagram. 12, 28, 36, 54, 72, 79, 135, 256, 270, 318, 371, 432
 - b) Which loop is empty? ______Explain why there is no number that belongs in that loop.



► Algebraic expressions contain variables such as *x* and *n*. *x* and *n* can represent any numbers you choose.

Here are some examples of algebraic expressions and what they mean.

x + 5:	Five more than a number
<i>n</i> – 3:	Three less than a number
3 – <i>n</i> :	Three subtract a number
5 <i>x</i> :	Five times a number
5n + 3:	Five times a number, then add 3; or three more than five times a number
$\frac{100}{n}$:	One hundred divided by a number
$\frac{n}{100}$:	A number divided by one hundred

- ▶ In the algebraic expression 7t + 2
 - 7 is the **numerical coefficient** of the variable.

2 is the **constant term**.

t is the variable.

► An algebraic expression can help you solve similar problems more efficiently. Once you know the algebraic expression, you can use it again, even if the numbers change.

Suppose you earn \$8 per hour.

For 3 hours, you earn: $3 \times \$8 = \24

For *t* hours, you earn: $t \times \$8 = 8t$ dollars

► To evaluate an expression means to substitute a number for the variable, then calculate the answer. To evaluate 2a - 5 for a = 7: Replace *a* with 7 in the expression 2a - 5. 2a - 5 = 2(7) - 5

$$= 14 - 5$$

HINT

If you see how to solve a problem using numbers, then you can write an algebraic expression for the problem. Replace the number that changes with a variable.



Recall that 2*a* means 2 x a. Use the order of operations. Multiply before subtracting.

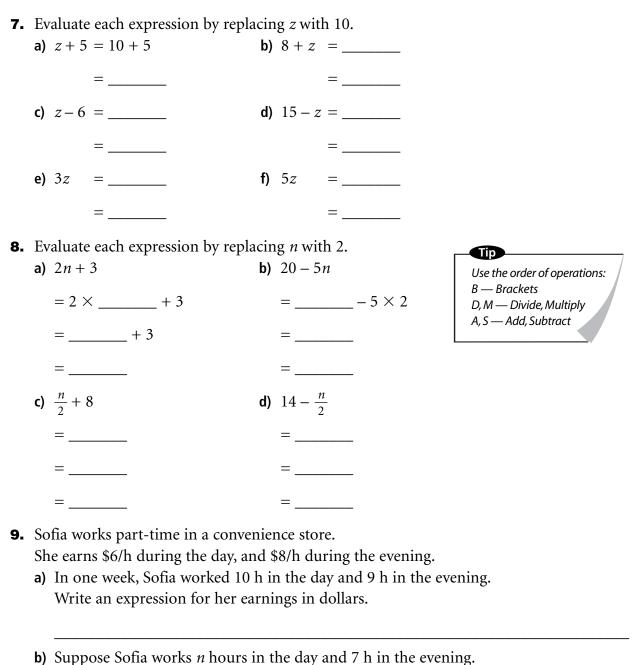
HINT

1. Match each algebraic expression with its meaning.

6 + x	Five less than a number
4 <i>n</i>	One more than double a number
1 + 2t	Five subtract a number
5 – <i>p</i>	Four times a number
<i>s</i> – 5	Three times a number subtract four
3g - 4	Six more than a number

2. Identify the numerical coefficient, the variable, and the constant term in each expression.

	a) 4 + 5 <i>s</i> Numerical coefficient:	Variable:	_ Constant term:
	b) $x + 7$		
	Numerical coefficient:	_ Variable:	_ Constant term:
	c) 9 <i>m</i> Numerical coefficient:	Variable:	_ Constant term:
з.	An algebraic expression has const	tant term 12, variable <i>t</i> ,	and numerical coefficient 8.
	What might the expression be?		
4.	Write an algebraic expression for Use the variable <i>n</i> .		
	a) Ten times a number	b) Double	a number
	c) A number divided by four	d) Six less	than a number
	e) Three more than ten times a r	number	
	f) Six less than ten times a numb	oer	
5.	A clerk earns \$12 an hour.		
	Find how much the clerk earns for	or each time.	
	a) 5 h work	b) 8 h work	c) p hours work
	5 ×=	=	=
6.	A car travels at an average speed	of 60 km/h.	
	Find how far it travels in each tin	ne.	
	a) 3 h	b) 5 h	c) x hours
	3 × =	=	=



i) Write an expression for her earnings in dollars.

ii) How much does Sofia earn when n = 5?

- c) Suppose Sofia works 9 h in the day and *m* hours in the evening.
 - i) Write an expression for her earnings in dollars.

ii) How much does Sofia earn when m = 11?

key to success

Evaluating algebraic expressions is an important skill. Carpenters, computer scientists, designers, electricians, and auto mechanics all use this skill to solve problems on the job.



➤ You can describe a number pattern using the term number.

Term number	1	2	3	4	5	6
Term	8	16	24	32	40	48

We can write an algebraic expression for the term when we know the term number. Each term is 8 times the term number.

Let *n* represent any term number.

Then the term is represented by $8 \times n$, or 8n.

When you compare or *relate* a variable to an expression that contains the variable, you have a *relation*.

The variable is *n*.

The expression is 8*n*.

The relation is: 8n is related to n

➤ The table and relation above can represent the total number of beats in a music score when there are 8 beats in each bar.

Number of bars of music	1	2	3	4	5	6
Total number of beats	8	16	24	32	40	48

You can use the relation to find the number of beats in 17 bars of music.

Substitute n = 17 in the expression 8n.

8n = 8(17)

```
= 136
```

There are 136 beats in 17 bars of music.

Practice

1. Complete each chart.

1						
Term number	1	2	3	4	5	6
Term	5		15		25	
				_		
Term number	1	2	3	4	5	6
Term	5		7		9	
Term number	1	2	3	4	5	6
Term	3		9		15	
	Term number Term number Term Term number	Term number1Term5Term number1Term5Term number1	Term number12Term57Term number12Term number12Term number12	Term number 1 2 3 Term 5 15 Term number 1 2 3 Term 5 7 Term number 1 2 3 Term 5 7 Term number 1 2 3	Term number 1 2 3 4 Term 5 15 15 Term number 1 2 3 4 Term 5 7 1 Term number 1 2 3 4 Term 5 7 1 Term number 1 2 3 4	Term number 1 2 3 4 5 Term 5 15 25 Term number 1 2 3 4 5 Term number 1 2 3 4 5 Term number 1 2 3 4 5 Term number 1 2 3 4 5

2. Every day, Ray rides his bike 12 km around Stanley Park. Complete the chart to show the total distance Ray travelled.

Number of days	1	2	3	4	5	6
Distance (km)	12					72

- Write a relation for the pattern rule for each pattern.
 Use the relation to find the 12th term.
 Let *n* represent any term number.
 - a) 6, 12, 18, 24, _____
 - **b)** 10, 11, 12, 13, _____
- **4.** a) Write a relation for the perimeter of a regular pentagon

with side length *n* centimetres.

- **b)** What is the perimeter of a regular pentagon with side length 9 cm?
- **5.** Ally is organizing an end-of-term party.

The cost to rent the hall is \$100. The cost of food is \$8 per person.

a) Write a relation for the total cost of the party, in dollars, for *n* people.

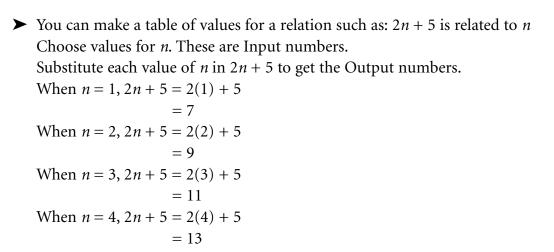
b) How much will the party cost if:

- i) 20 people attend?
- ii) 50 people attend? _____
- c) How does the relation in part a change in each case?i) The cost of food doubles.

ii) The cost of the food increases by \$2 per person.

- d) For each scenario in part c, find the cost when 40 people attend.
 - i) _____
 - ii) _____

1.5



Here is the table:

Input	Output
п	2 <i>n</i> + 5
1	7
2	9
3	11
4	13

➤ You can find a relation given its table of values.

	Input	Output	
1.0	1	2	
+1 5	2	6	< +4
+1 5	3	10	₹+4
+1	4	14	√ +4
+1 🦕	5	18	≁ ⁺⁴

Let *n* represent any Input number.

When n increases by 1, the Output number increases by 4.

This means that the expression for the Output numbers contains 4*n*.

So, compare the Output numbers to multiples of 4: 4, 8, 12, 16, 20, ...

Each Output number is 2 less than a multiple of 4.

So, the output is 4n - 2.

The table shows how 4n - 2 relates to *n*.

- - **b)** Complete the table. Use your results from part a.

Input n	Output $3n+1$
1	
2	
3	
4	
5	

2. Complete each table.

Explain how the Output number is related to the Input number.

a)	Input n	Output $n+5$	b)	Input b	Output 8 – b	c)	Input a	Output 6 + <i>a</i>
	1			1			1	
	2			2			2	
	3			3			3	
	4			4			4	
	5			5			5	

3. Complete each table.

a)

a)	Input d	Output $2d+3$
	1	
	2	
	3	
	4	
	5	

b)	Input f	Output $3f-2$
	1	
	2	
	3	
	4	
	5	

c)	Input h	Output 5 <i>h</i> + 1
	1	
	2	
	3	
	4	
	5	

4. Use algebra. Write a relation for each table.

b)

Input n	Output
1	2
2	3
3	4
4	5
5	6

II table.	
Input P	Output
1	0
2	1
3	2
4	3
5	4

c)	Input m	Output
	1	8
	2	16
	3	24
	4	32
	5	40

5. Use algebra. Write a relation for each table. Then extend each table 3 more rows.

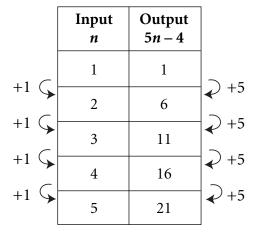
a)	Input r	Output
	1	4
	2	6
-	3	8
-	4	10
	5	12

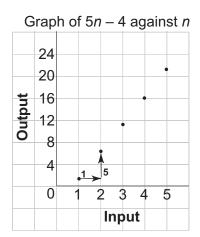
b)	Input s	Output
	1	2
	2	5
	3	8
	4	11
	5	14

c)	Input n	Output
	1	9
	2	14
	3	19
	4	24
	5	29



You can use a graph to show a relation.
 This table and graph show how 5n - 4 relates to n.





The scale on the *Output* axis is 1 square to 4 units. The points lie on a straight line, so the relation is linear. Both the table and the graph show that when the input increases by 1, the output increases by 5.

b)

Practice

1. Complete each table.

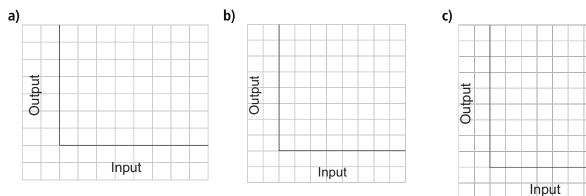
a)	Input n	Output $2n+8$
	1	10
	2	12
	3	14
	4	16
	5	
	6	
	7	

Input n	Output 5 <i>n</i> + 1
1	6
2	11
3	16
4	21
5	
6	
7	

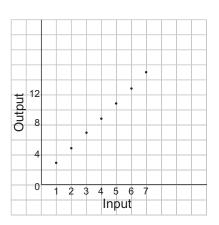
c)	Input n	Output 9 – <i>n</i>
	1	8
	2	7
	3	6
	4	5
	5	
	6	
	7	

2. Choose a suitable scale.

Graph each relation in question 1.



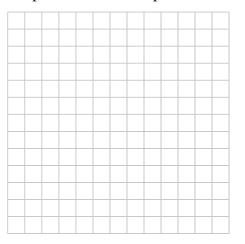
- **3.** Look at the graph at the right.
 - a) What is the output when the input is 1? _____
 - **b)** Which input gives an output of 13? _____
 - **c)** Extend the graph.
 - i) What is the output when the input is 8? _____
 - ii) What is the input when the output is 21? _____



4. a) Complete this table.

Input a	Output 5 <i>a</i> + 3
2	
4	
6	
8	
10	

b) Graph the relation in part a.



c) How does the graph illustrate the relation?

- **5.** The members of the student council wash cars to raise money for charity. The students charge \$3.00 per car.
 - a) Let *c* represent the number of cars washed.
 Write a relation to show how the money collected, in dollars, is related to the number of cars washed.

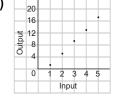
Number of cars	Money collected (\$)
10	
20	
30	
40	
50	

b) Complete this table to show the relation.

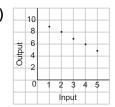
c) Graph the data.



- d) Describe the graph.

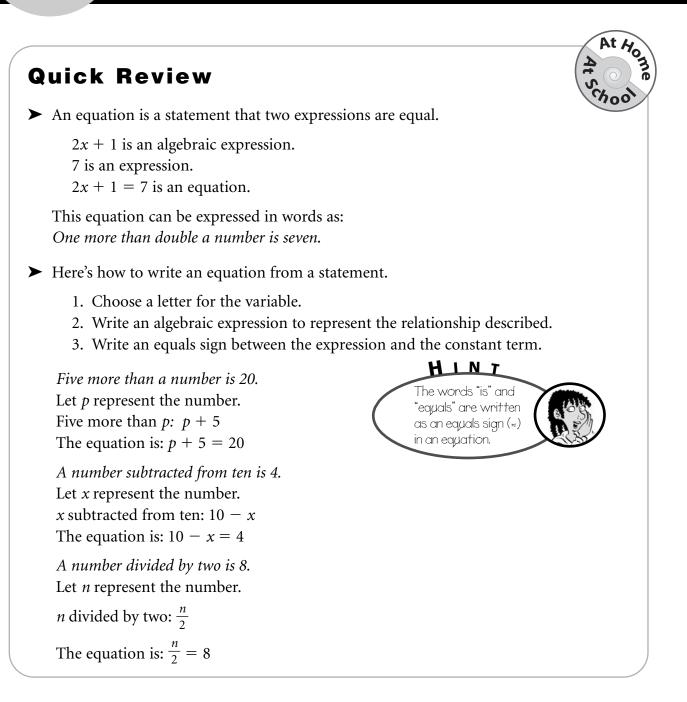


A. 10 − *n* relates to *n*



B. 3n + 5 relates to n

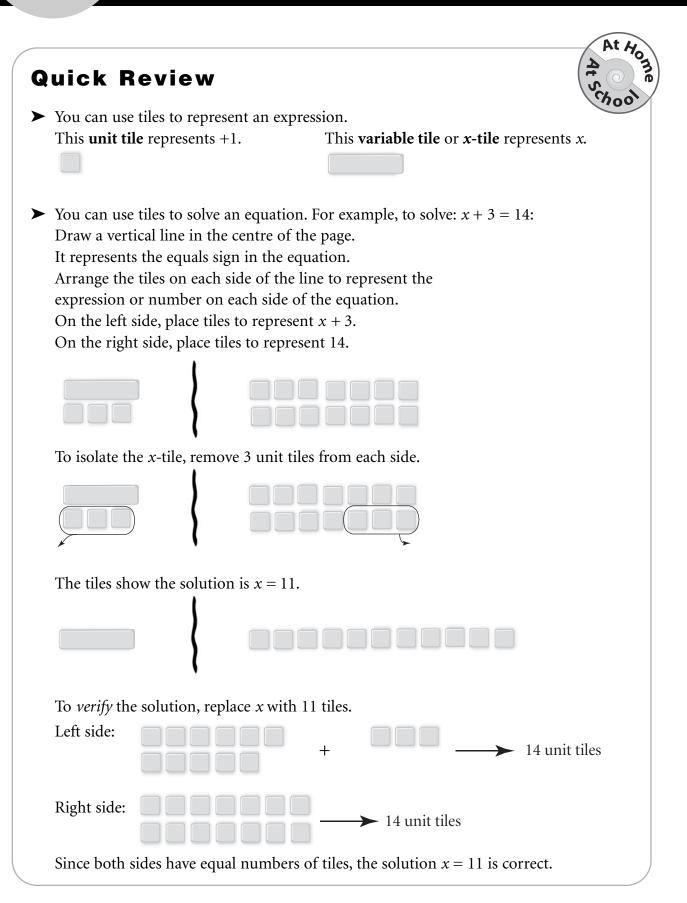
C. 4n - 3 relates to n



1. Match each sentence with an equation. The first one is done for you.

A number divided by three is 4.	20 - n = 6
Twenty subtract a number equals 6.	2n + 3 = 11
Nine subtract one-half a number is 6.	$\frac{n}{3} = 4$
Three added to double a number is 11.	$9 - \frac{n}{2} = 6$

- **2.** Write an equation for each sentence. Let *n* represent the number.
 - a) Eight less than a number is 2. $n ___ = ___$
 - **b)** One-half a number equals 5. _____
 - c) Four more than double a number is 20. _____
 - d) Six plus three times a number is 9.
- **3.** Write a sentence for each equation.
 - a) n 6 = 12
 - **b)** $\frac{x}{2} = 10$
 - c) 2p + 10 = 14
- **4.** Write an equation for each sentence. Let *x* represent the number.
 - a) Three more than a number is 12. _____
 - b) Three less than a number is 12.
 - c) Three times a number equals 12.
 - d) Three more than three times a number is 12.
 - e) Three subtracted from three times a number equals 12.
- **5.** Write an equation for each sentence.
 - a) The cost of 2 adult tickets at \$5 each and *n* child tickets at \$3 each is \$25.
 - b) William's age 4 years ago was 12. Let *a* years represent William's age now.
 - c) The perimeter of a square with side length *s* is 28.



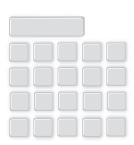
- **1.** Complete each algebraic expression.
 - a) A number increased by 3: x +_____
 - **b)** Two times a number: _____ *x*
 - c) Three more than 4 times a number: 4x +_____
 - **d)** Twelve less than a number: $_$ 12
- **2.** Match each picture to its equation.
 - a) x + 1 = 3

b)	x	+	2	=	4
----	---	---	---	---	---

c)	<i>x</i> +	20 =	12
~/	20 1	20	14



d) x + 12 = 20



	1	
	J	
1	1	
	J	
	1	
J	J	
	1	
	J.	

3. Zephyr had songs in his music player folder. He bought 7 more. Zephyr then had 10 songs. How many did he start with? Complete the solution for the equation: x + 7 = 10

Step 1		
Step 2		
Step 3		
The solut	10n 1s:	

4. An online book costs \$15.00 to upload to a computer. How many online books can be purchased for \$75.00?a) Write an equation to represent this problem.

b) Solve the equation to find how many online books can be purchased.

5. Erica is thinking of a number. She multiplies it by 2, then adds 5. The result is 19. Which number did Erica begin with?

- a) Write an equation to represent this problem.
- **b)** Solve the equation to find the number.

In Your Words

Here are some of the important mathematical words of this unit.

Build your own glossary by recording definitions and examples here. The first one is done for you.



List other mathematical words you need to know.

Unit Review

LESS	ON		
1.1	1.	a)	Circle the numbers that are divisible by 4.
			312 1407 204 3441 640 763
		b)	How do you know if a number is divisible by 4?
	2.	a)	Circle the numbers that are divisible by 2 and by 3.
			606 330 501 2466 492 9342
		b)	What other number are the circled numbers in part a divisible by?
			How do you know?
1.2	3.		hich numbers below are divisible by 8? Divisible by 5? ow do you know?
		a)	244:
		b)	160:
		,	100
		c)	315:
		-	
		d)	608:
	4.	Us	e your answers from question 3 to help you list all the factors of each number.
		a)	244:
		b)	160:
			315:
		u)	608:

LESSON

			te an algebi Three times					•						
			Five less tha											
		c)	Fwenty divi	ded by a	a nui	mber	:							
d) Seven more than four times a number:														
e	6.	Evaluate each expression for $n = 5$.												
a) $n + 7 = $ b) $10 - n = $ c) $2n + 3 =$											+ 3 =			
7	7. a) Zadie climbed four sets of stairs every minute for the Charity Stair Climb Fundation Complete this table. The pattern continues.											nb Fundrais		
			Time (minutes)	1	2	3	4	5	6	7	8			
			Sets of stai climbed	rs										
ę	9.	b) 8 Co1	nplete each	12, <u>-</u> table.										
		но [•] а)	w is each Or Input				lated	Inpu		ut nu Out		er: c) [Innut	Output
			n	3n+5				n		5 <i>n</i> -			Input n	Output 5 <i>n</i> – 3
			1					1					1	
			2					2					2	
			3					3					3	
			4					4				-	4	
			4		1		<u> </u>					4		1
			5					5					5	1

LESSON

a)

10. Use algebra. Write a relation for each table.

b)

Input m	Output
1	9
2	11
3	13
4	15
5	17

Input m	Output
1	9
2	16
3	23
4	30
5	37

c)	Input m	Output
	1	5
	2	12
	3	19
	4	26
	5	33

1.6 11. Graph each relation in question 10.

a)	_							
/		_	_	_	_	_		
		_	_	_	_	_		
	-	_	_	_	_	_	_	
	-	-	-	-	-	-		
	-	-	-	-	-	-		
	<u> </u>	-	-	-	-	-	_	



c)

- 1.7 **12.** Write an equation for each sentence. Let *n* represent the number.
 - a) Four times a number is sixteen.

b) Eight subtracted from four times a number is sixteen.

b)

c) Twelve more than four times a number is sixteen.

d) Thirty-two minus four times a number is sixteen.

13. Write an equation for each sentence. Let *n* represent the number.

a) Four less than a number is sixteen.

b) A number divided by five is ten.

c) Five more than three times a number is eleven.

1.8 14. Robin walked twice around a lake, plus an extra 3 km. Her pedometer showed that she had walked a total of 19 km. Write then solve an equation to find how far it is around the lake.