UNIT 5

Percent, Ratio, and Rate

The animal kingdom provides much interesting information. We use information to make comparisons. What comparisons can you make from these facts?

The amount of fruits and vegetables a grizzly bear eats a day represents about 25% of its body mass.

A Great Dane can eat up to 4 kg of food a day.

A cheetah can reach a top speed of 110 km/h.

A human can run at 18 km/h.

The heart of a blue whale is the size of a small car.

One in 5000 North Atlantic lobsters is born bright blue.

What You'll Learn

- Understand percents greater than or equal to 0%.
- Solve problems that involve percents.
- Understand ratios.
- Understand rates.
- Solve problems that involve ratios and rates.

Why It's Important

You use ratios and rates to compare numbers and quantities;

and to compare prices when you shop.

You use percents to calculate sales tax, price increases, and discounts.

Key Words

- percent increase
- percent decrease
- discount
- two-term ratio
- three-term ratio
- part-to-whole ratio
- part-to-part ratio
- equivalent ratios
- proportion
- rate
- unit rate

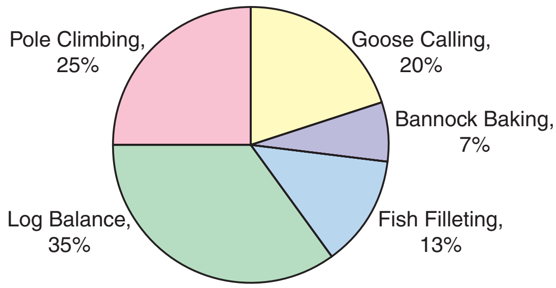
5.1 Relating Fractions, Decimals, and Percents

Focus Given a fraction, decimal, or percent, calculate the other two forms.

The students in a First Nations school were asked which of 5 events at the Northern Manitoba Trappers' Festival they would most like to attend. The circle graph shows the results.

Which event was the favourite? How do you know?
How else can you write that percent?

Northern Manitoba Trappers' Festival Event Preference



Investigate

Work with a partner.
The Grades 7 and 8 students in 2 schools in Winnipeg were asked which of these cultural or historical attractions they would most like to visit.

Attraction	Number of Students	
	School A	School B
Winnipeg Aboriginal Centre	48	18
Chinese Cultural Centre	30	24
Upper Fort Gary Gate	39	3
Gabrielle Roy House	33	15

➤ How many students were surveyed in each school?
➤ Which school had the greater percent of students choosing the Winnipeg Aboriginal Centre? The Chinese Cultural Centre? Upper Fort Gary Gate? Gabrielle Roy House?

Reflect & Share

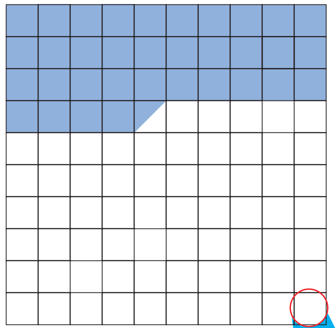
What strategies did you use to find the percents?
Compare your answers with those of another pair of classmates.
If the answers are different, how do you know which answers are correct?
At each school, what percent of students did not choose Upper Fort Gary Gate? How did you find out?

Connect

To write a fraction as a percent, we first write the fraction with a denominator that is a power of 10; such as 10, 100, 1000, or 10 000.
Some fractions cannot be written this way.
Then, we can use a calculator to divide.

A *power* of 10 is a product of any number of 10s.

➤ We can use a *hundred chart* to represent one whole, or 100%.
Each small square represents 1%.



We can describe the shaded part of the hundred chart in 3 ways:
as a percent, a decimal, and as a fraction.

There are $34\frac{1}{2}$ blue squares in 100 squares.

So, 34.5% of the squares are blue.

$\frac{1}{2} = 0.5$

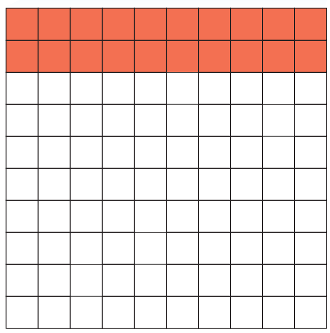
As a decimal: $\frac{34.5}{100} = 0.345$

As a fraction: Since the decimal has 3 digits after the decimal point, write a fraction with denominator 1000.

$$
\begin{aligned}
0.345 &= \frac{345}{1000} \\
&= \frac{345 \div 5}{1000 \div 5} \\
&= \frac{69}{200}
\end{aligned}
$$

**Reduce to simplest form.
5 is a factor of both the numerator and the denominator.**

➤ We can use a *hundredths chart* to represent 1%.
Each small square represents $\frac{1}{100}$ of 1%,
which we write as $\frac{1}{100}$%, or 0.01%.
To represent $\frac{1}{5}$ of 1%, or $\frac{1}{5}$% on the hundredths chart,
shade $\frac{1}{5}$ of the chart, which is 20 squares.
Since 1 small square is 0.01%, then 20 small squares are 0.20%, or 0.2%.



We can write this percent as a decimal.

$0.2\% = \frac{0.2}{100} = \frac{2}{1000} = 0.002$

Be careful not to confuse a decimal percent, such as 0.2%, with the decimal 0.2, which is 20%.

Example 1

Write each percent as a fraction and as a decimal.

a) 7% **b)** 7.75% **c)** $7\frac{1}{4}\%$

A Solution

a) $7\% = \frac{7}{100}$

$= 0.07$

b) $7.75\% = \frac{7.75}{100}$ Multiply the numerator and the denominator by 100.

$= \frac{775}{10\,000}$

$= 0.0775$

Write the fraction in simplest form.

$\frac{775}{10\,000} = \frac{775 \div 25}{10\,000 \div 25}$

$= \frac{31}{400}$

25 is a factor of both the numerator and the denominator. So, divide by 25.

c) $7\frac{1}{4}\% = \frac{7.25}{100}$

$\frac{1}{4} = 0.25$

$= \frac{725}{10\,000}$

$= 0.0725$

Write the fraction in simplest form.

$\frac{725}{10\,000} = \frac{725 \div 25}{10\,000 \div 25}$

$= \frac{29}{400}$

We can show each decimal in *Example 1* in a place-value chart.

Ones	Tenths	Hundredths	Thousandths	Ten Thousandths
0	0	7	0	0
0	0	7	7	5
0	0	7	2	5

Example 2

Write each fraction as a decimal and as a percent.

a) $\frac{5}{8}$ b) $\frac{5}{6}$ c) $\frac{5}{1000}$

A Solution

a) $\frac{5}{8}$ means $5 \div 8$. Use a calculator.

$\frac{5}{8} = 0.625$

$$0.625 = \frac{625}{1000}$$

Divide the numerator and the denominator by 10 to get an equivalent fraction with denominator 100.

$$= \frac{625 \div 10}{1000 \div 10}$$

$$= \frac{62.5}{100}$$

$$= 62.5\%$$

b) $\frac{5}{6} = 5 \div 6$ Use a calculator.

$= 0.8\overline{3}$

This is a repeating decimal.

Recall that the bar over the 3 indicates that the digit repeats.

To write an equivalent fraction with denominator 100, first write $0.8\overline{3}$ as $0.83\overline{3}$.

$$0.83\overline{3} = \frac{83.\overline{3}}{100}$$

$$= 83.\overline{3}\,\%$$

c) $\frac{5}{1000} = 5 \div 1000$

$= 0.005$

Divide the numerator and the denominator by 10 to get an equivalent fraction with denominator 100.

$$\frac{5}{1000} = \frac{5 \div 10}{1000 \div 10}$$

$$= \frac{0.5}{100}$$

$$= 0.5\%$$

We can use decimals or percents to compare two test marks when the total marks are different.

Example 3

Buffy had $23\frac{1}{2}$ out of 30 on her first math test.

She had $31\frac{1}{2}$ out of 40 on her second math test.

On which test did Buffy do better?

A Solution

Write each test mark as a percent.

First test:

$$23\frac{1}{2} \text{ out of } 30 = \frac{23.5}{30}$$ Divide. Use a calculator.

$$= 0.78\overline{3}$$

Since the decimal has 3 digits after the decimal point,
write a fraction with denominator 1000.

$$0.78\overline{3} = \frac{783.\overline{3}}{1000}$$

$$= \frac{783.\overline{3} \div 10}{1000 \div 10}$$ Divide the numerator and the denominator by 10 to get an equivalent fraction with denominator 100.

$$= \frac{78.\overline{3}}{100}$$

$$= 78.\overline{3}\ \%$$

Second test:

$$31\frac{1}{2} \text{ out of } 40 = \frac{31.5}{40}$$ Use a calculator.

$$= 0.7875$$

Since the decimal has 4 digits after the decimal point,
write a fraction with denominator 10 000.

$$0.7875 = \frac{7875}{10\,000}$$ Divide the numerator and the denominator by 100.

$$= \frac{7875 \div 100}{10\,000 \div 100}$$

$$= \frac{78.75}{100}$$

$$= 78.75\%$$

Since $78.75\% > 78.\overline{3}\ \%$, Buffy did better on the second test.

Discuss the ideas

1. How can we use a grid with 100 squares to show 100% and also to show 1%?

2. How can we use a grid with 100 squares to show 0%? Justify your answer.

3. Explain why $\frac{1}{5}$ and $\frac{1}{5}\%$ represent different numbers.

4. In *Example 1*, we simplified $\frac{775}{10\,000}$ by dividing the numerator and denominator by 25.

a) Why did we choose 25?

b) Could we have simplified the fraction a different way? Explain.

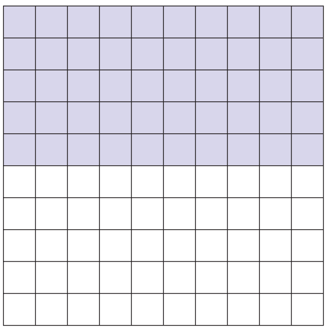
5. In *Example 3*, how could we solve the problem without finding percents?

Practice

Check

6. Each hundred chart represents 100%. What fraction of each hundred chart is shaded? Write each fraction as a decimal and as a percent.

a)



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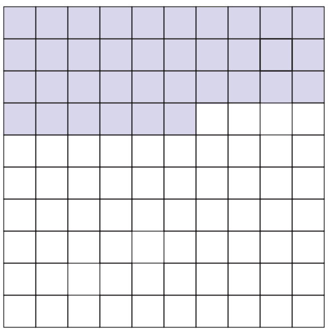

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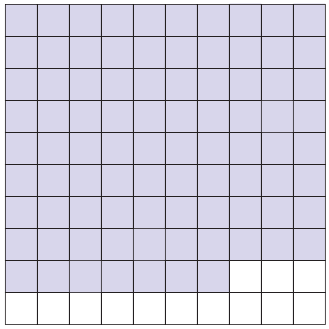

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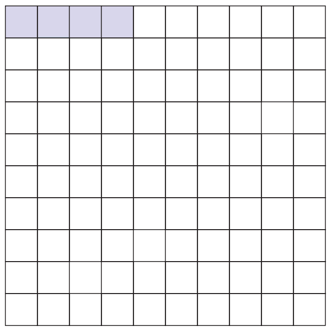
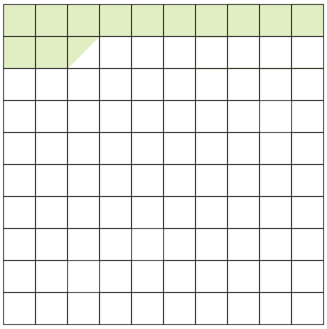


7. Write each percent as a fraction and as a decimal.

a) 3% **b)** 51% **c)** 98% **d)** 29%

8. Each hundred chart represents 100%. What fraction of each hundred chart is shaded? Write each fraction as a decimal and as a percent.

a)



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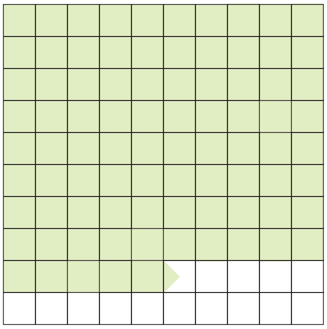
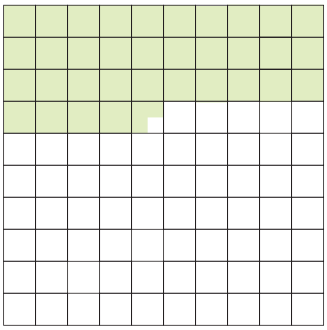


c)



Apply

9. Write each percent as a fraction and as a decimal.

a) 73.5% b) 21.25%

c) $8\frac{3}{4}$% d) $1\frac{1}{5}$%

10. Use a hundredths chart to represent 1%. Shade the chart to represent each percent.

a) 0.75% b) 0.4%

c) 0.07% d) 0.95%

11. Use a hundredths chart to represent 1%. Shade the chart to represent each percent.

a) 0.655% b) 0.0225%

c) $\frac{2}{3}$% d) $\frac{2}{5}$%

12. Write each percent as a fraction and as a decimal.

a) 0.25% b) 0.6%

c) 0.5% d) 0.38%

13. Write each fraction as a decimal and as a percent.

a) $\frac{2}{300}$ b) $\frac{18}{400}$

c) $\frac{7}{500}$ d) $\frac{8}{250}$

14. Write each decimal as a fraction and as a percent.

a) 0.345 b) 0.0023

c) 0.1825 d) 0.007

15. A hundredths chart represents 1%. Forty-five of its squares are shaded. Arjang says the shaded squares represent $\frac{45}{100}$. Fiona says the shaded squares represent 0.0045. Who is correct? Write to explain where the other student went wrong.

16. Vince scored 82.5% on a math test. Junita had 15 out of 18 on the same test. Who did better? How do you know?

17. Suppose you were asked to tutor another student.

a) **i)** How would you explain $\frac{5}{8}$ as a fraction?

ii) What real-life example could you use to help?

b) **i)** How would you explain $\frac{5}{8}$ as a quotient?

ii) What real-life example could you use to help?

18. **Assessment Focus** You will need 1-cm grid paper and coloured pencils.

a) Draw a 6-cm by 8-cm rectangle. Shade:

- $33.\overline{3}\%$ of the grid squares in the rectangle red
- 0.25 of the grid squares green
- $\frac{3}{8}$ of the grid squares blue

Explain how you did this.

b) What fraction of the rectangle is not shaded? Write this fraction as a decimal and as a percent.

c) Do you think you could have completed part a with a 6-cm by 9-cm rectangle? With a square of side length 7 cm? Explain.

19. A student council representative is elected from each homeroom class in the school. Joanna received 23 of 30 votes in her class. Kyle received 22 of 28 votes in his class. Who received the greater percent of votes, Joanna or Kyle? How did you find out?

20. A student used this strategy to write $6\frac{1}{4}\%$ as a fraction.

$$\begin{aligned} 6\tfrac{1}{4}\% &= 6.25\% \\ &= \frac{625}{100} \\ &= \frac{625 \div 25}{100 \div 25} \\ &= \frac{25}{4} \end{aligned}$$

a) Check the student's work. Is the strategy correct?

b) If your answer is yes, write the fraction as a decimal. If your answer is no, describe the error then correct it.

21. **Take It Further** Replace each $\square$ with <, >, or = to make each statement true.

a) $3.21 \ \square \ 321\%$

b) $1\frac{5}{8} \ \square \ 158\%$

c) $0.76 \ \square \ 7.6\%$

d) $0.9\% \ \square \ 0.9$

e) $0.\overline{3}\% \ \square \ \frac{1}{3}\%$

f) $125\% \ \square \ 1\frac{1}{4}$

Reflect

What did you know about fractions, decimals, and percents before you began this lesson? What do you know about fractions, decimals, and percents now?

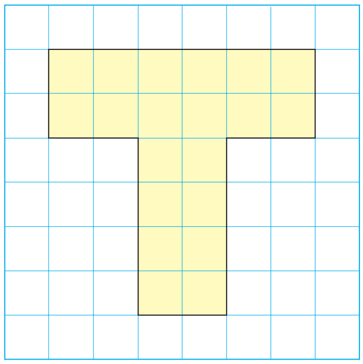
5.2 Calculating Percents

Focus Calculate percents from 0% to greater than 100%.

Have you ever used a photocopier to reduce or enlarge a picture?
To choose the size of the image picture, you select a percent.
Which percents might you choose if you want to reduce the picture?
Which percents might you choose if you want to enlarge the picture?

Investigate

Work with a partner.
Copy this shape.



➤ Redraw the shape so that each line segment is 150% of the length shown.

➤ Draw your own shape.
Choose a different percent between 100% and 200%.
Repeat the activity above.

Reflect & Share

Compare your drawings with those of another pair of classmates.
What strategies did you use to create your enlargements?
What do you notice about the lengths of corresponding line segments on the original shape and the enlarged shape?

Connect

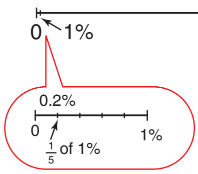
Recall that when the whole is 1.0, you know that:

100% = 1.0
10% = 0.10
1% = 0.01

We can extend the pattern to write percents less than 1% as decimals:

0.1% = 0.001
0.5% = 0.005

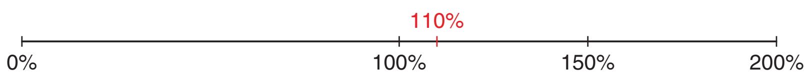
We can use number lines to show percents between 0% and 1%.
For example, this number line shows 0.2%.



We can also extend the pattern to write percents greater than 100% as decimals.

101% = 1.01
110% = 1.10, or 1.1
150% = 1.50, or 1.5
200% = 2.00, or 2.0

We can use a number line to show percents greater than 100%.



Percents greater than 100% are used by store owners
to calculate the prices of items they sell.
A store has to make a profit; that is, to sell goods
for more than the goods cost to buy.

A store manager buys merchandise from a supplier.
The price the manager pays is called the *cost* price.
The manager *marks up* the cost price to arrive at the *selling price* for the customer.
The markup is the *profit*.
Cost price + Profit = Selling price

Example 1

a) Write 210% as a decimal.

b) Shade hundred charts to show 210%.

A Solution

a) $210\% = \frac{210}{100}$

$= 2.10$, or 2.1

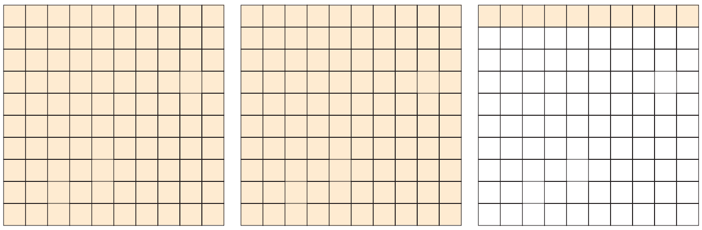
b) $210\% = 100\% + 100\% + 10\%$

Use a hundred chart to represent 100%.

To show 200%, shade all the squares in 2 hundred charts.

Each small square represents 1%.

So, to show 10%, shade 10 squares of a third hundred chart.



Example 2

The cost price of a winter coat is $80.

The selling price of the coat is 230% of the cost price.

What is the selling price of the coat?

Illustrate the answer with a number line.

A Solution

To find the selling price of the coat, find 230% of $80.

First, write 230% as a decimal.

$230\% = \frac{230}{100}$

$= 2.30$, or 2.3

Then, 230% of $\$80 = 2.3 \times \$80$

$= \$184$

The selling price of the coat is $184.

We can show this answer on a number line.

Example 3

In 2004, the population of First Nations people living on reserves in Alberta was 58 782.
About 0.28% of these people belonged to the Mikisew Cree band.

a) About how many people belonged to the Mikisew Cree band?
b) Estimate to check the answer is reasonable.
c) Illustrate the answer with a diagram.

A Solution

a) Find 0.28% of 58 782.
First write 0.28% as a decimal.

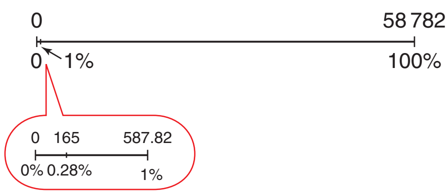
$$0.28\% = \frac{0.28}{100} \qquad \text{Multiply the numerator and the denominator by 100.}$$
$$= \frac{28}{10\,000}$$
$$= 0.0028$$

Then, 0.28% of 58 782 $= 0.0028 \times 58\,782$ Use a calculator.
$= 164.5896$

About 165 people belonged to the Mikisew Cree band.

b) 0.28% is approximately 0.25%.
0.25% is $\frac{1}{4}$%.
1% of 58 782 is: $0.01 \times 58\,782 = 587.82$
587.82 is about 600.
$600 \div 4 = 150$
This estimate is close to the calculated answer, 165.

c) To illustrate 0.28%, first show 1% on a number line.
Then, 0.28% is about $\frac{1}{4}$ of 1%.



Discuss the ideas

1. As a decimal, 100% = 1.
What decimals correspond to percents greater than 100%?
What decimals correspond to percents less than 1%?
2. In *Example 2*, how could you use the number line to find the profit?
3. In *Example 2*, how could you estimate to check the answer?

Practice

Check

4. A hundred chart represents 100%. Shade hundred charts to show each percent.
   a) 150% b) 212% c) 300% d) 198%

5. Write each percent as a decimal. Draw a diagram or number line to illustrate each percent.
   a) 120% b) 250% c) 475%
   d) 0.3% e) 0.53% f) 0.75%

6. Write each decimal as a fraction and as a percent.
   a) 1.7 b) 3.3 c) 0.003 d) 0.0056

7. The cost price of a baseball cap is $9. The selling price of the cap is 280% of the cost price. What is the selling price of the baseball cap? Illustrate the answer with a number line.

Apply

8. What does it mean when someone states, "She gave it 110%"?
   How can this comment be explained using math? Is it possible to give 110%? Explain.

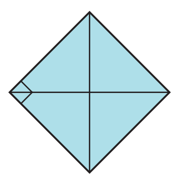
9. a) Describe two situations when a percent may be greater than 100%.
   b) Describe two situations when a percent may be between 0% and 1%.

10. a) Write each fraction as a percent.
    i) $\frac{1}{3}$ ii) $\frac{2}{3}$ iii) $\frac{3}{3}$
    iv) $\frac{4}{3}$ v) $\frac{5}{3}$ vi) $\frac{6}{3}$
    b) What patterns do you see in your answers in part a?
    c) Use these patterns to write each fraction as a percent.
    i) $\frac{7}{3}$ ii) $\frac{8}{3}$ iii) $\frac{9}{3}$
    iv) $\frac{10}{3}$ v) $\frac{11}{3}$ vi) $\frac{12}{3}$

11. a) Find each percent of the number. Draw a diagram to illustrate each answer.
    i) 200% of 360 ii) 20% of 360
    iii) 2% of 360 iv) 0.2% of 360
    b) What patterns do you see in your answers in part a?
    c) Use the patterns in part a to find each percent. Explain your work.
    i) 2000% of 360 ii) 0.02% of 360

12. A marathon had 618 runners registered. Of these runners, about 0.8% completed the race in under 2 h 15 min.
    a) How many runners completed the race in this time?
    b) Estimate to check your answer.

13. a) This shape represents 100%. Draw a shape that represents 375%.
    b) Repeat part a using a shape of your own choice.



14. The population of a small town in Alberta was 2600. The population increased by 5% one year and by 15% the next year. What was the town's population after the 2 years?

a) To solve this problem, Juan calculated the population after a 5% increase. He then used his number to find the population after a 15% increase. What was Juan's answer?

b) To solve this problem, Jeremy calculated the population after a 20% increase. What was Jeremy's answer?

c) Compare your answers to parts a and b. Are Juan and Jeremy's answers the same? If your answer is yes, explain why both strategies work. If your answer is no, who is correct? Justify your choice.

15. At the local theatre, 120 people attended the production of *Romeo and Juliet* on Friday. The attendance on Saturday was 140% of the attendance on Friday.

a) How many people went to the theatre on Saturday?

b) Estimate to check your answer is reasonable.

16. **Assessment Focus** During the 1888 Gold Rush, a British Columbia town had a population of about 2000. By 1910, the town had become a ghost town. The population was 0.75% of its population in 1888.

a) Estimate the population in 1910. Justify your estimate.

b) Calculate the population in 1910.

c) Find the decrease in population from 1888 to 1910. Show your work.

17. **Take It Further** Twenty boys signed up for the school play. The number of girls who signed up was 195% of the number of boys. At the auditions, only 26 girls attended. What percent of the girls who signed up for the play attended the auditions?

18. **Take It Further** At an auction, a painting sold for $148 500. This was 135% of what it sold for 3 years ago. What was the selling price of the painting 3 years ago? Justify your answer.

19. **Take It Further** The perimeter of a rectangular window is 280% of its length. The length of the window is 145 cm. What is the width of the window? Show your work.

Reflect

How do you find a percent of a number in each case?

- The percent is less than 1%.
- The percent is greater than 100%.

Use an example to explain each case. Include diagrams.

5.3 Solving Percent Problems

Focus Find the whole, when given a percent, and find the percent increase and decrease.

Investigate

Work with a partner.

Tasha conducted a survey of the students in her school.

- From the results, Tasha calculated that 60% of the students go to school by bus.
- Liam knows that 450 students go to school by bus. How can Liam use these data to find the number of students in the school?
- Tasha also found that 50% more students go by bus than walk or drive.
  About how many students walk or drive to school?

Sketch number lines to illustrate your work.

Reflect & Share

Compare your results with those of another pair of classmates. Discuss the strategies you used to solve the problems.

Connect

Grady is 13 years old and 155 cm tall.
His height at this age is about
90% of his final height.

To estimate Grady's final height:
90% of Grady's height is 155 cm.
So, 1% of his height is: $\frac{155 \text{ cm}}{90}$
And, 100% of his height is: $\frac{155 \text{ cm}}{90} \times 100 \doteq 172.2$ cm
So, Grady's final height will be about 172 cm.

When we know a percent of the whole, we divide to find 1%,
then multiply by 100 to find 100%, which is the whole.

Example 1

Find the number in each case.

a) 70% of a number is 63.

b) 175% of a number is 105.

A Solution

a) 70% of a number is 63.
So, 1% of the number is:
$\frac{63}{70} = 0.9$
And, 100% of the number is:
$0.9 \times 100 = 90$
The number is 90.

b) 175% of a number is 105.
So, 1% of the number is:
$\frac{105}{175} = 0.6$
And, 100% of the number is:
$0.6 \times 100 = 60$
The number is 60.

Example 2

a) A length of 30 cm increased by 40%. What is the new length?

b) A mass of 50 g decreased by 17%. What is the new mass?

A Solution

a) The length increased by 40%.
So, the increase in length is 40% of 30.
First, write 40% as a decimal.

$$40\% = \frac{40}{100}$$
$$= 0.4$$

Then, 40% of 30 $= 0.4 \times 30$
$= 12$

The length increased by 12 cm.
So, the new length is: 30 cm + 12 cm = 42 cm

b) The mass decreased by 17%.
So, the decrease in mass is 17% of 50.
First, write 17% as a decimal.

$17\% = \frac{17}{100}$, or 0.17

Then, 17% of 50 $= 0.17 \times 50$
$= 8.5$

The mass decreased by 8.5 g.
So, the new mass is: 50 g − 8.5 g = 41.5 g

Example 2
Another Solution

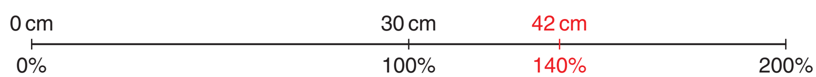
a) The length increased by 40%.
So, the new length is 100% + 40% = 140% of the original length.
Find 140% of 30.
First, write 140% as a decimal.

$140\% = \frac{140}{100}$, or 1.4

Then, 140% of 30 $= 1.4 \times 30$
$= 42$

The new length is 42 cm.

We can illustrate this answer on a number line.



b) The mass decreased by 17%.
So, the new mass is 100% − 17% = 83% of the original mass.
Find 83% of 50.
First, write 83% as a decimal.

$83\% = \frac{83}{100}$

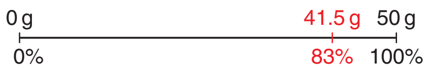
$= 0.83$

Then, 83% of 50 $= 0.83 \times 50$

$= 41.5$

The new mass is 41.5 g.

We can illustrate this answer on a number line.



Another type of problem involving percents is to find the **percent increase** or **percent decrease**.

Example 3

a) The price of a carton of milk in the school cafeteria increased from 95¢ to \$1.25.
What was the percent increase in price?

b) The price of a green salad decreased from \$2.50 to \$1.95.
What was the percent decrease in price?

A Solution

Write \$1.25 in cents: \$1.25 = 125¢

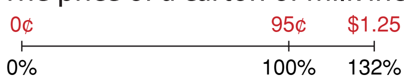
a) The increase in price was: 125¢ − 95¢ = 30¢
To find the percent increase, write the increase as a fraction of the original price: $\frac{30¢}{95¢}$

To write this fraction as a percent: $\frac{30}{95} \doteq 0.32$ Use a calculator.

$= \frac{32}{100}$

$= 32\%$

The price of a carton of milk increased by about 32%.



b) The decrease in price is: \$2.50 − \$1.95 = \$0.55, or 55¢
To find the percent decrease, write the decrease as a fraction of the original price: $\frac{55¢}{\$2.50} = \frac{55¢}{250¢}$

\$2.50 = 250¢

To write this fraction as a percent: $\frac{55}{250} = 0.22$

$= \frac{22}{100}$

$= 22\%$

Use a calculator.

The price of a green salad decreased by 22%.

Discuss the ideas

1. In *Example 2*, you learned two methods to find the new length and mass. Which method do you think you would prefer to use in the *Practice* questions that follow? Explain.

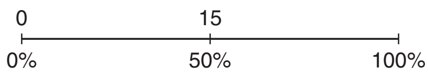
2. In *Example 3*, how could you check that the percent increase and percent decrease are correct?

Practice

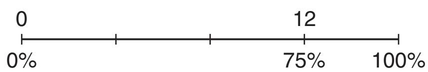
Check

3. Use the number line to find each number.

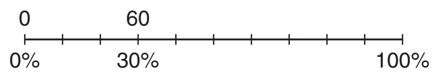
a) 50% of a number is 15.



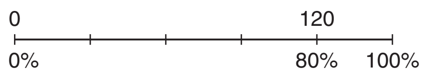
b) 75% of a number is 12.



c) 30% of a number is 60.



d) 80% of a number is 120.



4. Find the number in each case. Illustrate each answer with a number line.

a) 25% of a number is 5.
b) 75% of a number is 18.
c) 4% of a number is 32.
d) 120% of a number is 48.

5. Write each increase as a percent. Illustrate each answer with a number line.
   a) The elastic band stretched from 5 cm to 10 cm.
   b) The price of a haircut increased from \$8.00 to \$12.00.

6. Write each decrease as a percent. Illustrate each answer with a number line.
   a) The price of a book decreased from \$15.00 to \$12.00.
   b) The number of students who take the bus to school decreased from 200 to 150.

Apply

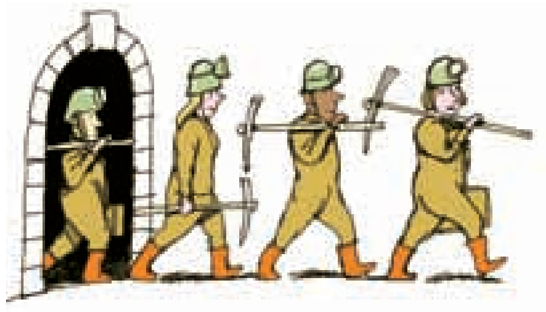
7. Find the whole amount in each case.
   a) 15% is 125 g.
   b) 9% is 45 cm.
   c) 0.8% is 12 g.

8. Write each increase as a percent. Illustrate each answer with a number line.
   a) The price of a house increased from \$320 000 to \$344 000.
   b) The area of forest in southwestern Yukon affected by the spruce bark beetle increased from 41 715 ha in 2003 to 99 284 ha in 2004.

One hectare (1 ha) is a unit of area equal to 10 000 m^2.

9. Write each decrease as a percent. Illustrate each answer with a number line.
   a) The price of gasoline decreased from 109.9¢/L to 104.9¢/L.
   b) The number of students in the class who listen to MP3 players decreased from 17 to 10.

10. There were about 193 000 miners in Canada in 1986. By 2001, the number of miners was 12% less. How many miners were there in 2001?



11. The world's tallest totem pole, known as the Spirit of Lekwammen, was raised on 04 August, 1994 at Victoria, BC, prior to the Commonwealth Games. The totem pole stood about 55 m tall. A local airport was concerned that seaplanes might hit it. So, in 1998 it was partially dismantled. It then stood about 12 m tall. Find the percent decrease in the height of the totem pole.

12. Olivia has 2 puppies, George and Jemma. Each puppy had a birth mass of 1.5 kg. At the end of Week 1, Jemma's mass was 15% greater than her birth mass. At the end of Week 2, Jemma's mass was 15% greater than her mass after Week 1. At the end of Week 2, George's mass was 30% greater than his birth mass.

a) What was each puppy's mass after Week 2?

b) Why are the masses in part a different?

13. **Assessment Focus** In 1990, the population of Calgary, Alberta, was about 693 000. The population increased by about 24% from 1990 to 2000. From 2000 to 2005, the population increased by about 11%.

a) In 2000, about how many people lived in Calgary?

b) In 2005, about how many people lived in Calgary?

c) Write the increase in population from 1990 to 2005 as a percent of the 1990 population.

d) Is your answer in part c 35%? Should the answer be 35%? Explain why or why not.

14. In 2004, the crime rate in a city was 15 194 crimes per 100 000 population. The crime rate decreased by 6% in 2005, and by 4% in 2006.

a) What was the crime rate at the end of 2006?

b) Is your answer to part a the same as a decrease in the crime rate of 10%? Why or why not?

15. On average, a girl reaches 90% of her final height when she is 11 years old, and 98% of her final height when she is 17 years old.

a) Anna is 11 years old. She is 150 cm tall. Estimate her height when she is 20 years old.

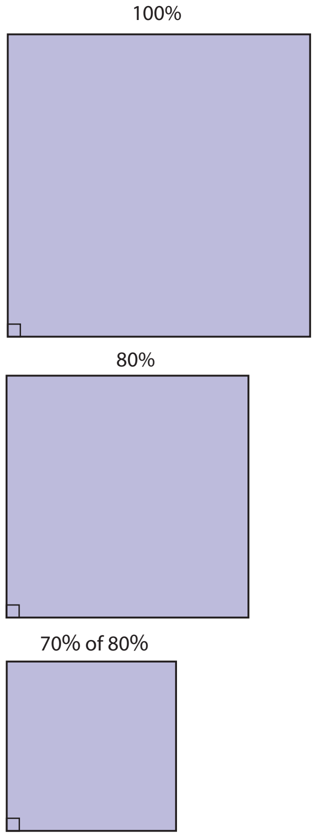
b) Raji is 17 years old. She is 176 cm tall. Estimate her height when she is 30 years old.

What assumptions do you make?

16. On average, a boy reaches 90% of his final height when he is 13 years old, and 98% of his final height when he is 18 years old. Use these data or the data in question 15 to estimate your height when you are 21 years old.
Explain any assumptions you make.
Show your work.

17. After a price reduction of 20%, the sale price of an item is $16. A student says, "So, the original price must have been 120% of the sale price."
Is this statement correct?
Justify your answer.

18. **Take It Further** A photocopier is used to reduce a square.
When the photocopier is set at 80%, the side length of the copy is 80% of its length on the original square.
Suppose the side length of the square is 10 cm. It is copied at 80%. The image square is then copied again at 70%.



a) What is the side length on the final image square?
b) What is the percent decrease in the side length of the square?
c) What is the area of the final image square?
d) What is the percent decrease in the area of the square?

19. **Take It Further** A box was $\frac{3}{4}$ full of marbles. The box fell on the floor. Thirty marbles fell out. This was 20% of the marbles in the box. How many marbles would a full box contain?

20. **Take It Further** Shen dug a 5-m by 15-m garden along one side of his rectangular lawn.
He says that this has reduced the area of his lawn by 25%. What are the dimensions of the remaining lawn?
Use a diagram to show your answer.
Describe the strategy you used to solve the problem. What assumptions do you make?

Reflect

What is the difference between a percent increase and a percent decrease?
Include examples in your explanation.

5.4 Sales Tax and Discount

Focus Investigate the use of percent in consumer math.

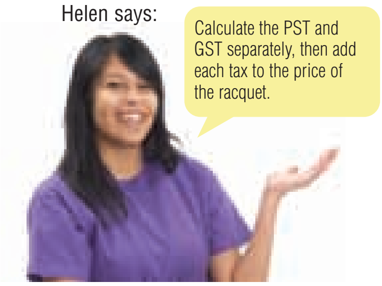
A sales tax is charged by the federal government and by most provincial governments. In 2007, the federal tax, the goods and services tax (GST), was 6%.
The provincial sales tax (PST) is set by each provincial or territorial government.
Some provinces have introduced a harmonized sales tax (HST), which combines both the PST and GST.

Province or Territory	Provincial Sales Tax	Goods and Services Tax
Northwest Territories (NT)	no PST	6%
Nunavut (NU)	no PST	6%
Yukon (YT)	no PST	6%
British Columbia (BC)	7%	6%
Alberta (AB)	no PST	6%
Saskatchewan (SK)	5%	6%
Manitoba (MB)	7%	6%
Ontario (ON)	8%	6%
Québec (QC)	7.5%	6%
Newfoundland and Labrador (NL)	14% HST	
Nova Scotia (NS)	14% HST	
New Brunswick (NB)	14% HST	
Prince Edward Island (PEI)	10%	6%

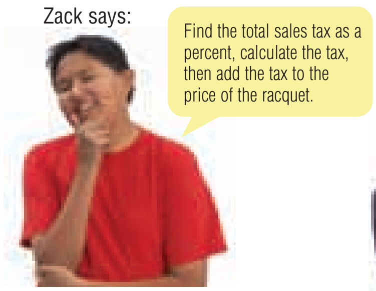
Investigate

Work with a partner.
Celine wants to purchase a tennis racquet in Winnipeg, Manitoba.
The racquet sells for $129.99.
To ensure she has enough money, Celine wants to calculate the final price of the racquet, including all taxes. She asks 3 friends for help.

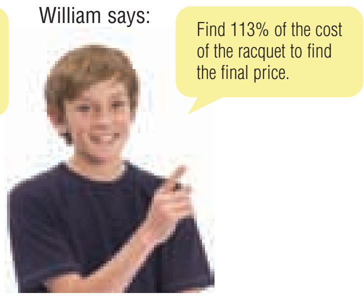
Helen says:



Zack says:



William says:



Use each method to calculate the final price of the racquet.

Reflect & Share

Compare your results with those of another pair of classmates.
What do you notice? Why do you think this happens?
Which of the three methods would you use? Justify your choice.

Connect

The selling price of an item is often the same throughout Canada. But the amount you pay depends on the province or territory where you buy the item.

Example 1

How much would you pay for this DVD in each place?

a) Nunavut **b)** Saskatchewan **c)** Nova Scotia

A Solution

a) In Nunavut, there is no PST.
The GST is 6%.
6% of $\$25.99 = 0.06 \times \$25.99$
$\doteq \$1.56$
So, the price you pay is: $\$25.99 + \$1.56 = \$27.55$

b) In Saskatchewan, the PST is 5% and the GST is 6%.
PST: 5% of $\$25.99 = 0.05 \times \$25.99$
$\doteq \$1.30$
GST: 6% of $\$25.99 = 0.06 \times \$25.99$
$\doteq \$1.56$
So, the price you pay is: $\$25.99 + \$1.30 + \$1.56 = \$28.85$

c) In Nova Scotia, the HST is 14%.
HST: 14% of $\$25.99 = 0.14 \times \$25.99$
$\doteq \$3.64$
So, the price you pay is: $\$25.99 + \$3.64 = \$29.63$

When an item is on sale for 20% off, we say that there is a **discount** of 20%.
A discount of 20% on an item means that you pay:
$100\% - 20\% = 80\%$ of the regular price

Example 2

A video game in Vancouver is discounted by 30%.
Its regular price is \$27.99.

a) Calculate the sale price of the video game before taxes.
b) Calculate the sale price of the video game including taxes.

A Solution

a) To find the amount of the discount, calculate 30% of \$27.99.

$30\% = \frac{30}{100} = 0.3$

So, 30% of \$27.99 $= 0.3 \times \$27.99$
$\doteq \$8.40$

The amount of the discount is \$8.40.
So, the sale price of the video game is: $\$27.99 - \$8.40 = \$19.59$

b) In Vancouver, the PST is 7% and the GST is 6%.
Calculate the taxes.
The PST is 7%.
7% of \$19.59 $= 0.07 \times \$19.59$
$\doteq \$1.37$
The GST is 6%.
6% of \$19.59 $= 0.06 \times \$19.59$
$\doteq \$1.18$
So, the sale price, including taxes, is: $\$19.59 + \$1.37 + \$1.18 = \$22.14$

Example 2
Another Solution

a) Find the sale price in one step.
The sale price of the video game is: 100% − 30%, or 70% of \$27.99
70% of \$27.99 $= 0.7 \times \$27.99$
$= \$19.59$
So, the sale price of the video game is \$19.59.

b) Find the sale price, including taxes, in one step.
The total sales tax is: 7% + 6% = 13%
\$19.59 is 100% of the sale price.
So, the sale price including taxes is 100% + 13%, or 113% of \$19.59.
113% of \$19.59 $= 1.13 \times \$19.59$
$\doteq \$22.14$
So, the sale price, including taxes, is \$22.14.

Example 3

The cost price of a backpack is $10.50.
This is 30% of the selling price.

a) What is the selling price of the backpack?

b) What does a customer pay for the backpack in Regina, Saskatchewan?

A Solution

a) 30% of the selling price is $10.50.
So, 1% of the selling price is: $\frac{\$10.50}{30}$
and 100% of the selling price is: $\frac{\$10.50}{30} \times 100 = 35$
The selling price of the backpack is $35.00.

b) The PST in Saskatchewan is 5%.
The GST is 6%.
So, the total sales tax is: $5\% + 6\% = 11\%$
So, the price you pay is 100% + 11%, or 111% of $35.00.

$$\begin{aligned} 111\% \text{ of } \$35.00 &= 1.11 \times \$35.00 \\ &= \$38.85 \end{aligned}$$

A customer in Regina pays $38.85 for the backpack.

Discuss the ideas

1. In *Example 1b*, the PST was calculated before the GST. Suppose these calculations were reversed. Do you think you would get the same answer? Explain.

2. In *Example 2*, the discount was calculated as $8.397. Why was the answer given as $8.40?

3. Suppose you bought a taxable item for $50. How much would you pay for it in your province or territory? What strategy would you use to find out?

Practice

Use a calculator when you need to.

Check

4. Suppose you are in Flin Flon, Manitoba. Find the PST on each item.

a) a pair of sunglasses that cost $15.00

b) a sunscreen that costs $8.99

c) a laser mouse that costs $21.99

5. Suppose you are in Fort Simpson, Northwest Territories. Find the GST on each item.

a) a digital camera that costs $89.97

b) a cordless phone that costs $24.97

c) a soccer ball that costs $17.99

6. Suppose you are in Victoria, British Columbia. Find the sales taxes on each item.

a) a package of light bulbs that costs $7.47

b) an inflatable raft that costs $32.99

c) a diving mask that costs $27.98

7. Suppose you are in Moose Jaw, Saskatchewan. For each item below:

a) Calculate the PST and GST.

b) Calculate the selling price including taxes.

i)

ii)

Apply

8. Suppose you are in Iqaluit, Nunavut. For each item below:

a) Calculate the discount.

b) Calculate the sale price before taxes.

c) Calculate the sale price including taxes.

i)

ii)

9. A new house was purchased for $304 000. After 3 years, its market value had increased by 28%. What was the market value of the house after 3 years?

10. **Assessment Focus** A video store offers these choices.
Choice A: 30% off each DVD with regular price $25.00
Choice B: Buy two DVDs for $40.00.
Which is the better deal for the customer? Justify your answer.

11. In a sale in Red Deer, Alberta, the price of a blow dryer is marked down.

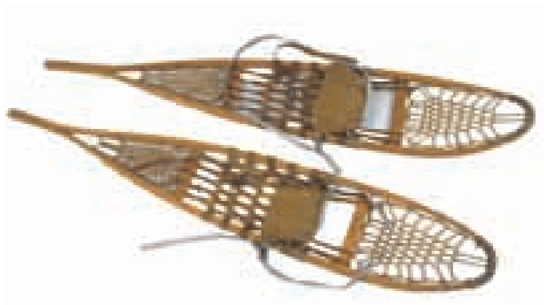
a) What is the percent decrease?
b) Calculate the sale price including taxes.

12. At the end of the summer, a gift store in Vancouver reduced the price of a souvenir T-shirt. The regular price of the T-shirt was $30. The T-shirt was reduced by 25%. The manager then put this sign in the window:

Rico told his mother that the T-shirt was now half its regular price. Was Rico correct? Justify your answer.

13. During a 20%-off sale, the sale price of an MP3 alarm clock radio was $35.96. What was the regular price of the radio?

14. The regular price of a pair of snowshoes in Brandon, Manitoba, is $129.99. The price of the snowshoes is marked down by 13%. Abbott says that the sale price including taxes will be $129.99 because the discount and taxes cancel each other out. Is Abbott's reasoning correct? Justify your answer.



15. The price of a hair straightener in Fredericton, New Brunswick, is reduced by $28.38. This is a discount of 33%.
a) What is the regular price of the hair straightener?
b) What is the sale price of the hair straightener including taxes?

16. Anika wants to buy a snowboarding helmet. The original price is $75.00. It is on sale for 30% off. Anika will pay 13% sales tax. A customer behind Anika in the line suggested that it would be cheaper if the 30% discount was subtracted before calculating the sales tax. Another customer said it would be cheaper if the 13% sales tax was added before the discount was subtracted. Who is correct? Show how you found out.

17. **Take It Further** Two identical lacrosse sticks are on sale at two stores in Dawson City, Yukon. At Strictly Sports, the stick is on sale for 15% off its regular price of $45.99. At Sport City, the stick is on sale for 20% off its regular price of $49.99. Which store offers the better deal? How much would you save? Show your work.

18. **Take It Further** For a promotion, a store offers to pay the sales taxes on any item you buy. You are actually paying taxes, but they are calculated on a lower price.
Suppose you buy an item for $100.
The store pays the 14% sales tax.

a) What is the true sale price of the item?
b) How much tax are you really paying?

19. **Take It Further** A pair of shoes in a clearance store went through a series of reductions. The regular price was $125. The shoes were first reduced by 20%. Three weeks later, the shoes were reduced by a further 20%.
Later in the year, the shoes were advertised for sale at $\frac{3}{4}$ off the current price. Sean wants to buy the shoes. He has to pay 11% sales tax.

a) Sean has $40.00.
Can Sean buy the shoes?
How did you find out?
b) If your answer to part a is yes, how much change does Sean get?

20. **Take It Further** A skateboard in a store in Charlottetown, PEI, costs $39.99. It is on sale for 30% off.
The taxes are 16%. What is the sale price of the skateboard including taxes?
A student says, "$39.99 – 30% discount + 16% taxes is the same as calculating $39.99 – 14%."
Is the student's reasoning correct?
Explain.

Reflect

Describe two different methods to calculate the sales tax on an item.
Which method do you prefer?
Which method is more efficient?
Include an example using each method.

Mid-Unit Review

LESSON

5.1

1. Write each percent as a fraction and as a decimal.
a) 60% **b)** 9.75% **c)** 97.5%

2. Use a hundredths chart to represent 1%. Shade a hundredths chart to show each percent.
a) 0.12% **b)** $\frac{4}{5}$% **c)** 0.65% **d)** $\frac{1}{10}$%

3. Write each decimal as a fraction and as a percent.
a) 0.18 **b)** 0.006 **c)** 0.875 **d)** 0.0075

5.2

4. Write each percent as a decimal. Draw a diagram to illustrate each percent.
a) 145% **b)** 350% **c)** 0.44% **d)** 0.2%

5. Jon reported his mark on a test as 112%. Is this mark possible? Give an example to support your answer.

6. A local school raised $5687.50 for the Terry Fox Foundation. Hua raised 0.8% of this total. How much money did Hua raise?

5.3

7. Find the number in each case.
a) 15% of a number is 3.
b) 160% of a number is 80.

8. Meryl sold 8% of the tickets to the school play. Meryl sold 56 tickets. How many tickets were sold in all?

9. Write this decrease as a percent. Illustrate this percent on a number line.
The attendance fell from 9850 to 8274.

5.4

10. The regular price of an ink cartridge is $32. It is on sale in Kelowna, BC, for 20% off. Calculate the sale price including taxes.

11. During a 30%-off sale in Portage la Prairie, Manitoba, the sale price of a memory stick was $27.99.
a) What was the regular price of the memory stick?
b) What was the regular price of the memory stick including taxes?
c) What is the sale price of the memory stick including taxes?
d) How much do you save by buying the memory stick on sale?

12. Jessica gets a 15% employee discount on everything she buys from Fashion City, in Whitehorse, Yukon. The regular price of a pair of jeans in this store is $69. They have been reduced by 30%. How much will Jessica pay for the jeans including taxes?

5.5 Exploring Ratios

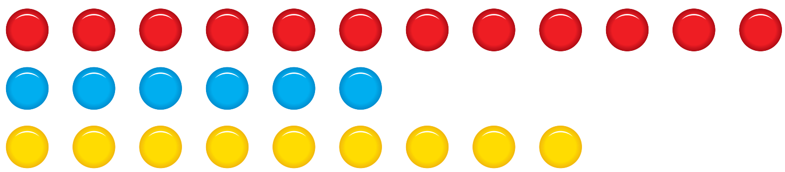
Focus Use models and diagrams to investigate ratios.

There are different ways to compare numbers.
Look at these advertisements.

How are the numbers in each advertisement compared?
Which advertisement is most effective? Why do you think so?

Investigate

Work with a partner.



Compare the number of blue counters to the number of yellow counters.
How many different ways can you compare the counters?
Write each way you find.

Reflect & Share

Share your list with another pair of classmates.
Add any new comparisons to your list.
Talk about the different ways you compared the counters.

Connect

Here is a collection of sports balls.

➤ We can use a **two-term ratio** to compare one part of the collection to the whole collection.
There are 7 basketballs compared to 20 balls.
The ratio of basketballs to all the balls is 7 to 20, which is written as 7:20.

This is a **part-to-whole** ratio.

We can write a part-to-whole ratio as a fraction.
The ratio of basketballs to all the balls is $\frac{7}{20}$.

A part-to-whole ratio can also be written as a percent: $\frac{7}{20} = \frac{35}{100} = 35\%$
So, 35% of the balls are basketballs.

➤ We can use a two-term ratio to compare one part of the collection to another part of the collection.
There are 5 golf balls compared to 8 tennis balls.
The ratio of golf balls to tennis balls is written as 5 to 8, or 5:8.
We cannot write this ratio in fraction form because the ratio is not comparing one part to the whole.

This is a **part-to-part** ratio.

➤ We can use a **three-term ratio** to compare the three types of balls.
There are 5 golf balls to 8 tennis balls to 7 basketballs.
We can write this as the ratio:
5 to 8 to 7, or 5:8:7

Example

At a class party, there are 16 boys, 15 girls, and 4 adults.
Show each ratio as many different ways as you can.

a) boys to girls
b) boys to girls to adults
c) adults to total number of people at the party

A Solution

a) There are 16 boys to 15 girls.
This is a part-to-part ratio.
So, the ratio of boys to girls is: 16 to 15, or 16:15

b) There are 16 boys, 15 girls, and 4 adults.
So, the ratio of boys to girls to adults is: 16 to 15 to 4, or 16:15:4

c) The total number of people is $16 + 15 + 4 = 35$.
There are 4 adults to 35 people.
This is a part-to-whole ratio.
So, the ratio of adults to total number of people is:
4 to 35; 4:35, $\frac{4}{35}$, or about 11.4%

Discuss the ideas

1. What is the difference between a part-to-whole ratio and a part-to-part ratio?
2. In the *Example*, explain how the part-to-whole ratio of 4:35 can be written as about 11.4%.
3. In the *Example*, why were the ratios of boys to girls and boys to girls to adults *not* written in fraction form?

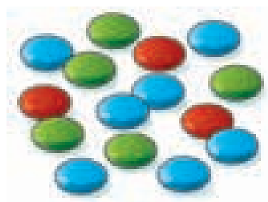
Practice

Check

4. Write each part-to-whole ratio as a fraction.
   a) 5:8 **b)** 12:16 **c)** 4:9 **d)** 24:25

5. Write each part-to-whole ratio as a percent.
   a) 19:20 **b)** 12:15 **c)** 3:8 **d)** 5:6

6. Look at the candy-covered chocolates below. Explain what each ratio means.
   a) 3:5
   b) 7:5
   c) 5:15
   d) 3:5:7
   e) 3:12



7. Look at the golf balls below. Write each ratio in two different ways.

a) orange golf balls to the total number of golf balls

b) white golf balls to the total number of golf balls

c) yellow golf balls to pink golf balls

d) yellow golf balls to white golf balls to orange golf balls

Apply

8. The ratio of T-shirts to shorts in Frank's closet is 5:2.

a) Write the ratio of T-shirts to the total number of garments.

b) Write the ratio in part a as a percent.

9. a) Write a part-to-part ratio to compare the items in each sentence.

i) A student had 9 green counters and 7 red counters on his desk.

ii) In a dance team, there were 8 girls and 3 boys.

iii) A recipe called for 3 cups of flour, 1 cup of sugar, and 2 cups of milk.

b) Write a part-to-whole ratio for the items in each sentence in part a.

Express each ratio as many ways as you can.

10. a) What is the ratio of boys to girls in your class?

b) What is the ratio of girls to boys?

c) What is the ratio of boys to the total number of students in your class? Write the ratio as a percent.

d) Suppose two boys leave the room. What is the ratio in part c now?

11. A box contains 8 red, 5 green, 2 orange, 3 purple, 1 blue, and 6 yellow candies.

a) Write each ratio.

i) red:purple

ii) green:blue

iii) purple:blue:green

iv) orange and yellow:total candies

b) Suppose 3 red, 2 green, and 4 yellow candies were eaten. Write the new ratios for part a.

12. Suppose you were asked to tutor another student.

a) How would you explain $\frac{2}{7}$ as a ratio?

b) What real-life example could you use to help?

13. a) Draw two different diagrams to show the ratio 3:5.

b) Draw a diagram to show the ratio 7:1.

c) Draw a diagram to show the ratio 5:2:4.

d) Why can you draw 2 diagrams in part a, but not in parts b and c?

14. **Assessment Focus** Patrick plans to make macaroni salad. The recipe calls for:

3 cups of cooked macaroni
3 cups of sliced oranges
2 cups of chopped apple
1 cup of chopped celery
2 cups of mayonnaise

a) What is the total amount of ingredients?

b) What is each ratio?

  i) oranges to apples

  ii) mayonnaise to macaroni

  iii) apples to mayonnaise to celery

c) What is the ratio of apples and oranges to the total amount of ingredients? Write this ratio as a fraction and as a percent.

d) Patrick uses 2 cups of oranges instead of 3. What are the new ratios in parts b and c?

e) Write your own ratio problem about this salad. Solve your problem.

15. Look at the words below.

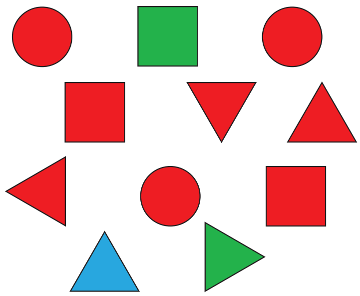
- ratio
- discount
- decimal
- problem
- percent
- increase
- taxes
- number

Which words represent the ratio 2:5? Explain what the ratio means each time.

16. **Take It Further** Maria shares some cranberries with Jeff. Maria says, "Two for you, three for me, two for you, three for me …" Tonya watches. At the end, she says, "So Jeff got $\frac{2}{3}$ of the cranberries." Do you agree with Tonya? Give reasons for your answer.

17. **Take It Further**

a) Create four different ratios using these shapes.



b) How can you change one shape to create ratios 2:5 and 7:3? Explain.

18. **Take It Further** Choose a vowel-to-consonant ratio. Find 3 words that represent this ratio.

Reflect

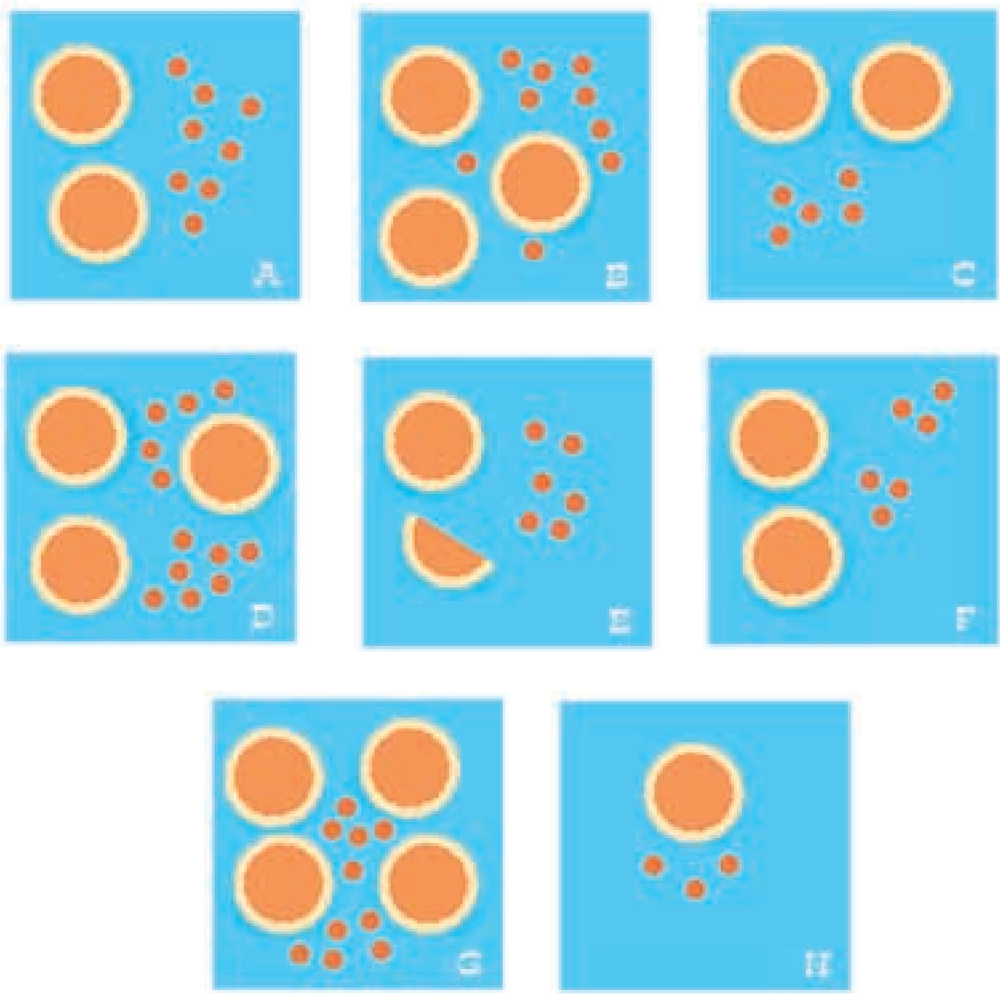
Give 3 examples from your classroom that can be represented by the ratio 1:1.

5.6 Equivalent Ratios

Focus Write equivalent ratios.

Investigate

Work with a partner.
Which cards have the same ratio of pepperoni pieces to pizzas?

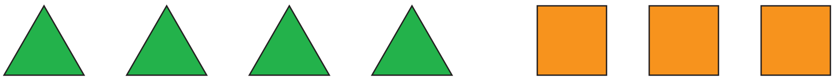


Reflect & Share

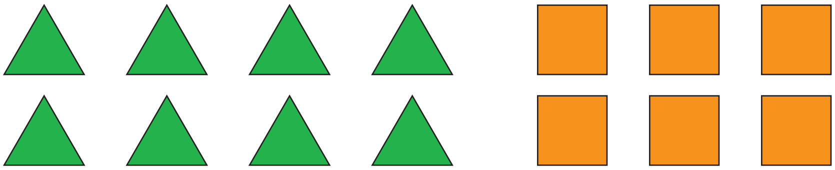
Share your answers with another pair of classmates.
What strategies did you use to identify the same ratios?
Why do you think your answers are correct?
What patterns do you see?

Connect

The ratio of triangles to squares is 4:3.
That is, for every 4 triangles, there are 3 squares.



The ratio of triangles to squares is 8:6.
That is, for every 8 triangles, there are 6 squares.



The ratios 8:6 and 4:3 are called **equivalent ratios**.
Equivalent ratios are equal.
8:6 = 4:3

An equivalent ratio can be formed by multiplying or dividing the terms of a ratio by the same number.

➤ We can show this with the terms of the ratios in rows.

$\div 4$ $\div 2$ $\times 2$ $\times 3$ $\times 4$ $\times 5$

1	2	**4**	8	12	16	20
0.75	1.5	**3**	6	9	12	15

$\div 4$ $\div 2$ $\times 2$ $\times 3$ $\times 4$ $\times 5$

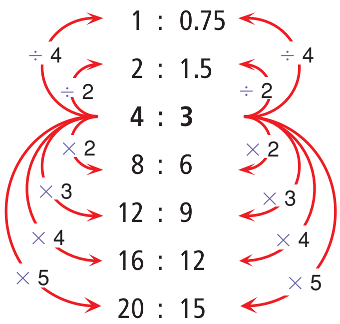
Note that:

$3 \div 2$ is $\frac{3}{2} = 1.5$

and

$3 \div 4$ is $\frac{3}{4} = 0.75$

➤ We can show this with the terms of the ratios in columns.

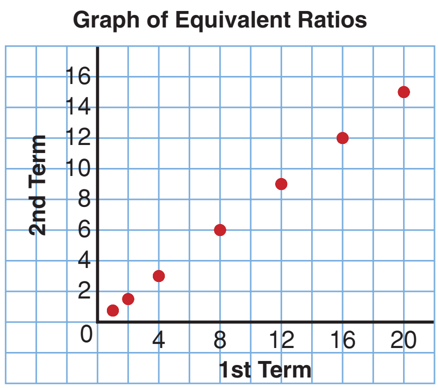


The equivalent ratios are:
1:0.75; 2:1.5; 4:3, 8:6; 12:9; 16:12; 20:15

➤ These ratios can be plotted as points on a grid.
Show the equivalent ratios in a table.

1st term	1	2	4	8	12	16	20
2nd term	0.75	1.5	3	6	9	12	15

Plot the points.
The points lie on a straight line.



➤ When we divide the terms in a ratio by their *greatest common factor*, we write the ratio in *simplest form*.

A ratio is in simplest form when its terms have no common factors.

To write 24:16 in simplest form:

24:16 The greatest common factor is 8.

$= \frac{24}{8}:\frac{16}{8}$ Divide by 8.

$=$ 3:2

So, 24:16 and 3:2 are equivalent ratios.
The ratio 3:2 is in simplest form.

Example 1

Write 3 ratios equivalent to 2:5.

A Solution

2:5

Multiply each term by 2.

$(2 \times 2):(5 \times 2)$

$= 4:10$

Multiply each term by 3.

$(2 \times 3):(5 \times 3)$

$= 6:15$

Multiply each term by 4.

$(2 \times 4):(5 \times 4)$

$= 8:20$

Three equivalent ratios are 4:10, 6:15, and 8:20.

Example 2

Write 3 ratios equivalent to 36:6.

A Solution

36:6

Divide each term by 2.

$\frac{36}{2}:\frac{6}{2}$

$= 18:3$

Divide each term by 3.

$\frac{36}{3}:\frac{6}{3}$

$= 12:2$

Divide each term by 6.

$\frac{36}{6}:\frac{6}{6}$

$= 6:1$

Three equivalent ratios are 18:3, 12:2, and 6:1.

Example 3

Construction kits come in different sizes.
The regular kit contains 120 long rods, 80 short rods, and 40 connectors. List 3 other kits that could be created with the same ratio of rods and connectors.

A Solution

For the regular kit, the ratio of long rods to short rods to connectors is 120:80:40.
For each new kit, find an equivalent ratio.
For Kit A, divide each term by 2.

$$120:80:40 = \frac{120}{2}:\frac{80}{2}:\frac{40}{2} = 60:40:20$$

Kit A has 60 long rods, 40 short rods, and 20 connectors.

For Kit B, divide each term by 4.

$$120:80:40 = \frac{120}{4}:\frac{80}{4}:\frac{40}{4}$$
$$= 30:20:10$$

Kit B has 30 long rods, 20 short rods, and 10 connectors.

There are many more possible kits.
These are just 3 of them.

For Kit C, multiply each term by 2.

$$120:80:40 = (120 \times 2):(80 \times 2):(40 \times 2)$$
$$= 240:160:80$$

Kit C has 240 long rods, 160 short rods, and 80 connectors.

Discuss the ideas

1. On pages 270 and 271, you saw equivalent ratios with terms written horizontally and vertically. Which representation is easier to follow? Justify your answer.
2. a) How do you know when a ratio is in simplest form?
   b) Which of the ratios in *Example 1* and *Example 2* is in simplest form? How do you know?
3. In *Example 1* and *Example 2*, which other equivalent ratios could be written for each ratio?
4. Are these ratios equivalent? Explain.
   3:2   2:3

Practice

Check

5. Write 3 ratios equivalent to each ratio. Use tables to show your work.

a) 1:2 b) 2:3 c) 1:4

6. Write 3 ratios equivalent to each ratio. Use tables to show your work.

a) 3:4 b) 14:4 c) 24:25

7. Write 3 ratios equivalent to each ratio. Use tables to show your work.

a) 1:3:6 b) 12:5:7 c) 24:4:8

8. Write each ratio in simplest form.

a) 5:15 b) 6:9
c) 3:12:18 d) 110:70:15

9. Write a ratio, in simplest form, to compare the items in each sentence.

a) In a class, there are 32 chairs and 8 tables.
b) In a parking lot, there were 4 American cars and 12 Japanese cars.
c) A paint mixture is made up of 6 L of blue paint, 2 L of yellow paint, and 1 L of white paint.
d) A stamp collection contains 12 Canadians stamps, 24 American stamps, and 9 Asian stamps.

10. Find the missing number in each pair of equivalent ratios.

a) 2:7 and $\square$:28
b) 5:12 and 25:$\square$
c) $\square$:24 and 5:3
d) 3:$\square$:11 and 30:70:110

Apply

11. a) Find pairs of equivalent ratios:

2:3:4	9:12:15
8:5:4	1:2:3
3:2:1	16:10:8
3:6:9	6:9:12
5:8:4	3:4:5

b) Tell how you know they are equivalent.

12. In a class library, 3 out of 4 books are non-fiction. The rest are fiction.

a) How many non-fiction books could there be? How many fiction books could there be?
b) How many different answers can you find for part a? Which answers are reasonable? Explain.

13. The official Canadian flag has a length-to-width ratio of 2:1.

Doreen has a sheet of paper that measures 30 cm by 20 cm. What are the length and width of the largest Canadian flag Doreen can draw? Sketch a picture of the flag.

14. **Assessment Focus**

a) Draw a set of counters to represent each ratio.

i) red:blue = 5:6
ii) blue:green = 3:4
iii) red:blue:green = 10:12:16

How many different ways can you do this? Record each way you find.

b) Draw one set of counters that satisfies all 3 ratios. How many different ways can you do this?

15. **Take It Further** Are these ratios equivalent? How do you know?

a) 16:30 and 28:42
b) 27:63 and 49:21
c) 56:104:88 and 42:78:66
d) 20:70:50 and 30:105:75

16. **Take It Further** There are 32 students in a Grade 8 class.
The ratio of girls to boys is 5:3.

a) How many boys are in the class?
b) How many girls are in the class?

How did you find out?

17. **Take It Further** Find the missing number in each pair of equivalent ratios.

a) 10:35 and $\square$:42
b) 36:78 and $\square$:182
c) $\square$:15 and 68:85
d) 49:$\square$:63 and 84:36:108

18. **Take It Further** A quality control inspector finds that 8 out of 9 batteries from the production line meet or exceed customer requirements.

a) Write each ratio.

i) number of batteries that passed to number of batteries that failed
ii) number of batteries that passed to the number of batteries tested
iii) number of batteries that failed to the number of batteries tested

b) After some changes to the production line, the number of batteries that did not meet or exceed customer requirements decreased by one-half. Rewrite the ratios in part a to reflect these changes.

Reflect

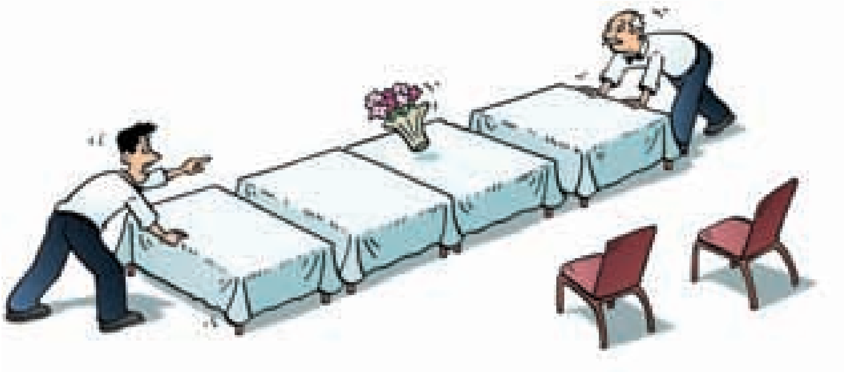
Choose a ratio. Use pictures, numbers, or words to show how to find two equivalent ratios.

Explaining Your Thinking

Explaining how you solved a problem helps you and others understand your thinking.

Solve this problem.

A restaurant has square tables.
Each table seats 4 people.
For large parties and banquets,
the tables are put together in rows.
How many people can be seated
when 6 tables are put together?
When 20 tables are put together?



Recall the problem-solving strategies you know.

Strategies

- **Make a table.**
- **Use a model.**
- **Draw a diagram.**
- **Solve a simpler problem.**
- **Work backward.**
- **Guess and test.**
- **Make an organized list.**
- **Use a pattern.**
- **Draw a graph.**
- **Use logical reasoning.**

Strategies for Success

When you have found the solution to a problem, write a few sentences to explain how you solved the problem. These sentences should help someone else understand how you solved the problem.

Here is one way to describe your thinking:

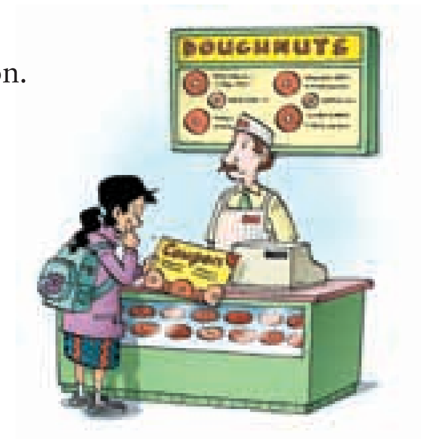
- Describe the problem.
- Describe the strategies you used—even the ones you tried that did not lead you to a solution.
- Describe the steps you took.
- Describe how you know your answer is correct.

Solve these problems.
For each problem, write a few sentences to describe your thinking.

1. There are 400 students at a school.
Is the following statement true? Explain.
There will always be at least 2 students in the school whose birthdays fall on the same day of the year.

2. Camden has a custard recipe that needs:
6 eggs, 1 cup of sugar, 750 mL of milk, and 5 mL of vanilla
He has 4 eggs. Camden adjusts the recipe to use the 4 eggs.
How much of each other ingredient will Camden need?

3. Lo Choi wants to buy a dozen doughnuts. She has a coupon.
This week, the doughnuts are on sale for \$3.99 a dozen.
If Lo Choi uses the coupon, each doughnut is \$0.35.
Should Lo Choi use the coupon?
Justify your answer.



Triple Play

HOW TO PLAY

Your teacher will give you a copy of *Triple Play* game cards.

Triple Play Cards

10:15	28:35	24:8	12:16
8:14	35:28	15:21	27:33
40:35	9:21	14:18	24:18
7:14	33:45	15:6	10:25
7:35	42:30	6:18	24:15
56:32	27:81	22:99	21:6
12:42	30:18	27:15	18:30
21:36	33:21	14:49	13:91

1. Cut out the cards, then shuffle them. Place the cards face down in a pile.
2. Player A turns over 1 card. He says the ratio in simplest form, then says 2 equivalent ratios.
   One point is awarded for the ratio in simplest form.
   One point is awarded for each equivalent ratio.
3. Player B takes a turn.
4. Players continue to take turns.
   The player with the higher score after 6 rounds wins.

YOU WILL NEED

A set of *Triple Play* game cards; paper; pencil

NUMBER OF PLAYERS

2

GOAL OF THE GAME

To get the greater number of points

TAKE IT FURTHER

Create a set of cards with the ratios in simplest form.
Combine the cards you created with the game cards.
Shuffle the cards, then deal 8 cards to each player.
The goal of the game is to collect pairs of cards that show equivalent ratios. Students pick and discard a card each round.
The player with more equivalent ratios after 8 rounds wins.

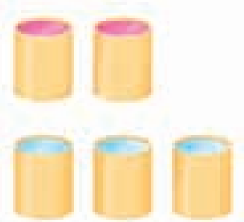
5.7 Comparing Ratios

Focus Use different strategies to compare ratios.

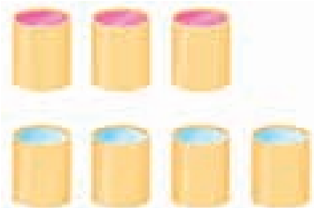
Investigate

Work with a partner.

Recipe A for punch calls for 2 cans of concentrate and 3 cans of water.



Recipe B for punch calls for 3 cans of concentrate and 4 cans of water.



In which recipe is the punch stronger?
Or, are the drinks the same strength?
Explain how you know.

Reflect & Share

Compare your answer with that of another pair of classmates.
Compare strategies.
If your answers are the same, which strategy do you prefer? Would there be a situation when the other strategy would be better? Explain.
If your answers are different, find out which answer is correct.

Connect

Erica makes her coffee with 2 scoops of coffee to 5 cups of water.

Jim makes his coffee with 3 scoops of coffee to 7 cups of water.

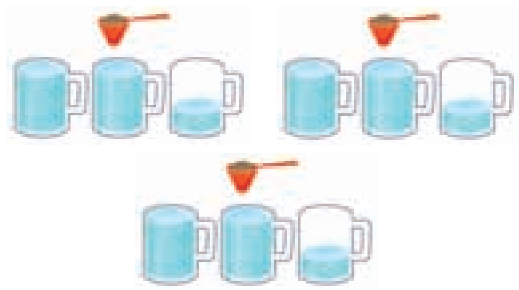
Here are two strategies to find out which coffee is stronger.

➤ Draw a picture.
Find how much water is used for 1 scoop of coffee.

Erica

1 scoop of coffee to
$2\frac{1}{2}$ cups of water

coffee to water = $1:2\frac{1}{2}$



Jim

1 scoop of coffee to
$2\frac{1}{3}$ cups of water

coffee to water = $1:2\frac{1}{3}$

Since $2\frac{1}{3}$ is less than $2\frac{1}{2}$, Jim uses less water to 1 scoop of coffee.
So, Jim's coffee is stronger.

➤ Use equivalent ratios.
Find how much coffee is used for the same amount of water.
Write equivalent ratios with the same second term.
Then compare the first terms.

Erica

Coffee		Water
2	:	5
4	:	10
6	:	15
8	:	20
10	:	25
12	:	30
14	:	**35**

Since 2:5 = 14:35,
Erica uses 14 scoops of
coffee to 35 cups of water.

Jim

Coffee		Water
3	:	7
6	:	14
9	:	21
12	:	28
15	:	**35**

Since 3:7 = 15:35,
Jim uses 15 scoops of
coffee to 35 cups of water.

Jim uses more coffee for the same amount of water. So, Jim's coffee is stronger.

Another way to compare ratios is to write equivalent ratios, with 1 as the second term.

Example 1

The recommended seeding on a package of grass seed is 200 g per 9 m^2. Carey spread 150 g over 6.5 m^2. Is this more than, equal to, or less than the recommended seeding?
How do you know?

A Solution

Write each ratio with second term 1.
For each ratio, divide each term by the second term.
Then, use a calculator to write each fraction as a decimal.

Package

Grass seed:Area

$200:9 = \frac{200}{9}:\frac{9}{9}$ Divide by 9.

$= 22.\overline{2}:1$

The package recommended $22.\overline{2}$ g of grass seed per 1 m^2.

Carey

Grass seed:Area

$150:6.5 = \frac{150}{6.5}:\frac{6.5}{6.5}$ Divide by 6.5.

$\doteq 23.1:1$

Carey used about 23.1 g of grass seed per 1 m^2.

Since $23.1 > 22.\overline{2}$, Carey spread more seed than the package recommended.

Example 1 involved part-to-part ratios.
Some problems are solved using part-to-whole ratios.

Example 2

a) Write each part-to-part ratio as a part-to-whole ratio.

i) 2:3 ii) 4:3

b) Write each part-to-whole ratio on part a in fraction form.
Which part-to-whole ratio is greater?

A Solution

a) i) 2:3
$= 2:(2 + 3)$
$= 2:5$

ii) 4:3
$= 4:(4 + 3)$
$= 4:7$

b) Write each part-to-whole ratio in fraction form.

i) 2:5
$= \frac{2}{5}$

ii) 4:7
$= \frac{4}{7}$

Since $\frac{2}{5} < \frac{1}{2}$ and $\frac{4}{7} > \frac{1}{2}$, then $\frac{4}{7} > \frac{2}{5}$
So, 4:7 is greater than 2:5.

We can also write ratios as percents to compare them.

Example 3

A contractor brought 2 shades of yellow paint for his clients to see. Shade 1 is made by mixing 5 cans of yellow paint with 3 cans of white paint. Shade 2 is made by mixing 7 cans of yellow paint with 4 cans of white paint.
The clients want the lighter shade.
Which shade should they choose?
What assumptions do you make?

A Solution

Write the ratio of cans of yellow paint to cans of white paint for each shade.

Shade 1	**Shade 2**
5:3	7:4

The lighter shade will have less yellow paint.
Assume all the cans are the same size.
Write part-to-whole ratios for the number of cans of yellow paint to the total number of cans.

Shade 1

$5:(3+5) = 5:8$
$= \frac{5}{8}$
$= 62.5\%$

Shade 2

$7:(4+7) = 7:11$ Write each ratio as a fraction.
$= \frac{7}{11}$ Write each fraction as a percent.
$= 63.\overline{63}\,\%$

Shade 1 has 62.5% yellow paint.
Shade 2 has $63.\overline{63}\,\%$ yellow paint.

Since $62.5\% < 63.\overline{63}\,\%$, Shade 1 is the lighter shade.
The clients should choose Shade 1.

Discuss the ideas

1. You have seen these strategies to compare ratios.
- Draw a picture.
- Use equivalent ratios.
- Make a term 1.
- Use percents.

a) Which strategy do you prefer?
b) Could you always use that strategy? Explain.

2. **a)** How could you use ratios with second term 1 to solve the coffee problem in *Connect*?
b) Which strategy is most efficient for the coffee problem? Justify your answer.
- Draw a picture.
- Use equivalent ratios.
- Make the second term 1.

3. In *Example 3*, explain how the fraction $\frac{7}{11}$ was written as the percent $63.\overline{63}\,\%$.

Practice

Check

4. Write each ratio with first term 1.
a) 3:12 **b)** 5:40 **c)** 8:56
d) 9:81 **e)** 33:99 **f)** 22:132

5. Write each ratio with second term 1.
a) 16:4 **b)** 55:11 **c)** 144:12
d) 120:24 **e)** 91:13 **f)** 96:8

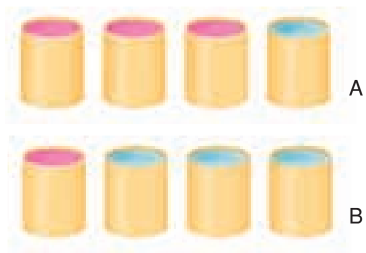
6. The principal is deciding which shade of blue to have the classrooms painted. One shade of blue requires 3 cans of white paint mixed with 4 cans of blue paint. Another shade of blue requires 5 cans of white paint mixed with 7 cans of blue paint.
a) Which mixture will give the darker shade of blue? Explain.
b) Which mixture will require more white paint?

7. In a hockey skills competition, Olga scored on 3 of 5 breakaways. Tara scored on 5 of 7 breakaways. Whose performance was better?
a) To find out, write each ratio as a fraction.
b) How can you use common denominators to help you solve the problem? Explain.

Apply

8. A chicken farmer in Manitoba compares the numbers of brown eggs and white eggs laid in 2 henhouses.
The chickens in Henhouse A lay 6 brown eggs for every 10 white eggs.
The chickens in Henhouse B lay 3 brown eggs for every 9 white eggs.
Which henhouse produces more white eggs? What assumptions do you make?

9. The concentrate and water in each picture are mixed.



Which mixture is stronger: A or B?
Draw a picture to show your answer.

10. In a basketball game, Alison made 6 of 13 free shots. Nadhu made 5 of 9 free shots. Who played better? Explain. Use two different methods to compare the ratios.

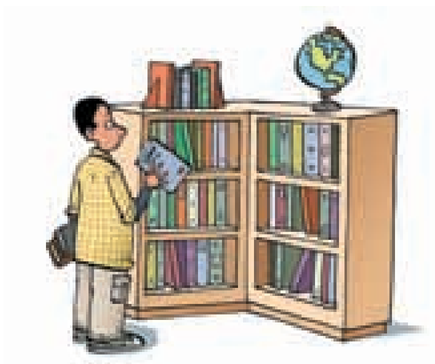
11. Two different groups at a summer camp have pizza parties. The Calgary Cougars order 2 pizzas for every 3 campers. The Alberta Antelopes order 3 pizzas for every 5 campers.
a) Which group gets more pizza per person? How do you know?
b) Could you use percent to find out? Why or why not?

12. Rick is comparing two recipes for oil and vinegar salad dressing. Recipe A calls for 150 mL of vinegar and 250 mL of oil. Recipe B calls for 225 mL of vinegar and 400 mL of oil. Which salad dressing will have a stronger vinegar taste? How did you find out?

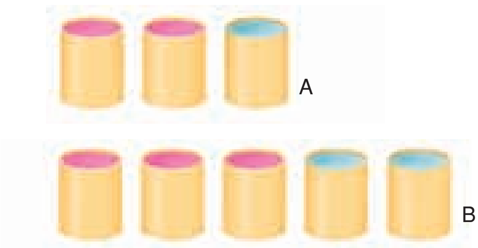
13. **Assessment Focus** The ratio of fiction to non-fiction books in Ms. Arbuckle's class library is 7:5. The ratio of fiction to non-fiction books in Mr. Albright's class library is 4:3. Each classroom has 30 non-fiction books.

a) Which room has more fiction books? How many more?

b) What percent of the books in each class is non-fiction?



14. Look at the sets of cans of concentrate and water.



a) What is the ratio of concentrate to water in A and in B?

b) Explain how you could add concentrate or water to make both ratios the same. Draw a picture to show your answer.

15. Drew has 3 shades of paint.

Shade	Red (drops)	Yellow (drops)
A	4	12
B	3	15
C	2	3

a) Write the ratio of red to yellow paint in each shade. Draw a picture to represent each ratio.

b) For each ratio in part a, write an equivalent ratio that uses 1 drop of red paint.

c) Which shade will have the most red? How do you know?

d) Which shade will have the most yellow? How do you know?

16. Two cages contain white mice and brown mice. The ratio of white mice to brown mice in Cage A is 5:6. The ratio of white mice to brown mice in Cage B is 7:5. Which cage contains more brown mice?
Marcel says, "Since $\frac{6}{11}$ is greater than $\frac{1}{2}$, and $\frac{5}{12}$ is less than $\frac{1}{2}$, Cage A contains more brown mice." Is Marcel's reasoning correct? Explain.

17. **Take It Further** All aircraft have a glide ratio. A glide ratio compares the distance moved forward to the loss in altitude. For example, an aircraft with a glide ratio of 7:1 will move forward 7 m for every 1 m of altitude lost.
Two hang-gliders start at the same altitude. Glider A has a glide ratio of 14:3. Glider B has a glide ratio of 15:4. When the gliders reach the ground, which one will have covered the greater horizontal distance? How did you find out?

18. **Take It Further** Two boxes contain pictures of hockey and basketball players. The boxes contain the same number of pictures.
In one box, the ratio of hockey players to basketball players is 4:3. In the other box, the ratio is 3:2.

a) What could the total number of pictures be?
b) Which box contains more pictures of hockey players? Draw a picture to show your answer.

19. **Take It Further** Valerie uses a powdered iced tea mix.
She always uses 1 more scoop of mix than cups of water. Valerie uses 5 scoops of mix for 4 cups of water.
To make more iced tea, she uses 7 scoops of mix for 6 cups of water.
To make less iced tea, she uses 3 scoops of mix for 2 cups of water.

a) Will all these mixtures have the same strength?
b) If your answer to part a is yes, explain your reasoning.
If your answer to part a is no, which mixture will be the strongest? Justify your answer.

Reflect

Look back at your answer to *Discuss the Ideas*, question 1a.
Now that you have completed *Practice*, is your answer to this question unchanged? Explain.

5.8 Solving Ratio Problems

Focus Set up a proportion to solve a problem.

We can use ratios when we change a recipe.

Investigate

Work with a partner.
Here is a recipe for an apple pie that serves 6 people.

500 mL flour
200 mL margarine
500 g sliced apples
125 g sugar

Toby has only 350 g of sliced apples.
How much of each other ingredient does Toby need to make the pie?
How many people will Toby's pie serve? Explain.

Reflect & Share

Compare your answers with those of another pair of classmates.
What strategies did you use to solve the problems?

Connect

In a recycle drive last week at Island Middle School, Mr. Bozyk's Grade 8 class collected bottles and recycled some of them.
The ratio of bottles recycled to bottles collected was 3:4.
This week, his class collected 24 bottles.
Mr. Bozyk told the students that the ratio of bottles recycled to bottles collected is the same as the ratio the preceding week.
How can the students find how many bottles are recycled this week?

Last week's ratio of bottles recycled to bottles collected is 3:4.
Let r represent the number of bottles recycled this week.
This week's ratio of bottles recycled to bottles collected is r:24.
These two ratios are equivalent.
So, $r:24 = 3:4$
A statement that two ratios are equal is a **proportion**.

To find the value of r, write a ratio equivalent to 3:4, with second term 24.
Since $4 \times 6 = 24$, multiply each term by 6.

$3:4 = (3 \times 6):(4 \times 6)$
$\quad = 18:24$

So, $r:24 = 18:24$
So, $r = 18$
18 bottles were recycled.

When we solve problems this way, we use proportional reasoning.

Example 1

Find the value of each variable.

a) $5:x = 40:56$ **b)** $49:35 = 14:n$

A Solution

a) $5:x = 40:56$
Since $5 < 40$, we divide to find x.
Think: What do we divide 40 by to get 5?

$\div 8$
$5:x = 40:56$

Divide 56 by the same number to get x.
$x = 56 \div 8$
$\quad = 7$
So, $x = 7$

b) $49:35 = 14:n$
Since $14 < 49$, we divide to get n.
Think: What do we divide 49 by to get 14?
$49:35 = 14:n$
$\div ?$

We cannot find a factor, so we simplify the ratio first.

$49:35 = \frac{49}{7}:\frac{35}{7}$
$\quad = 7:5$

Now, $7:5 = 14:n$
Think: What do we multiply 7 by to get 14?
$7:5 = 14:n$
$\times 2$

Multiply 5 by the same number to get n.
$n = 5 \times 2$
$\quad = 10$
So, $n = 10$

We can use proportions to solve ratio problems.

Example 2

This is a photo of a father and his daughter.
In the photo, the father's height is 8 cm and the daughter's height is 6 cm.
The father's actual height is 1.8 m.
What is the actual height of his daughter?

A Solution

In the photo, the ratio of the height of the father to the height of the daughter is 8:6.

Let h represent the actual height of the daughter, in centimetres.
Then, the ratio of the actual height of the father to the actual height of the daughter is $1.8:h$.
$1.8 \text{ m} = 180 \text{ cm}$
So, the ratio is $180:h$.
The two ratios are equivalent.
Then, $180:h = 8:6$

To find the value of h, write a ratio equivalent to 8:6, with first term 180.
Since 8 does not divide into 180 exactly, simplify the ratio 8:6 first.

$$8:6 = \frac{8}{2}:\frac{6}{2}$$
$$= 4:3$$

Write a ratio equivalent to 4:3, with first term 180.
Since $4 \times 45 = 180$, multiply each term by 45.

$$4:3 = (4 \times 45):(3 \times 45)$$
$$= 180:135$$

So, $180:h = 180:135$
So, $h = 135$
The daughter's actual height is 135 cm, or 1.35 m.

For some proportions, a suitable equivalent ratio cannot be written by multiplying or dividing each term by the same number.

Example 3

A bike is in fourth gear.
When the pedals turn 3 times,
the rear wheel turns 7 times.
When the pedals turn twice,
how many times does the rear wheel turn?

A Solution

The ratio of pedal turns to rear wheel turns is 3:7.
When the pedal turns twice, let the number of rear wheel turns be n.
The ratio of pedal turns to rear wheel turns is $2:n$.
These ratios are equal.
Then $3:7 = 2:n$
We cannot easily write a ratio equivalent to 3 to 7 with first term 2.
We look for a multiple of the first terms; that is, a multiple of 2 and 3.
The least multiple is 6.
We multiply each ratio to get the first term 6.

$$(3 \times 2):(7 \times 2) = (2 \times 3):(n \times 3)$$
$$6:14 = 6:3n$$

When we multiply each ratio in a proportion by a different number, the new ratios are still equivalent.

The first terms are equal, so the second terms must also be equal.
That is, $3n = 14$

$$\frac{3n}{3} = \frac{14}{3}$$
$$n = \frac{14}{3}, \text{ or } 4\tfrac{2}{3}$$

When the pedals turn twice, the rear wheel turns $4\frac{2}{3}$ times.

Discuss the ideas

1. In *Example 1*, how could we check that our answers are correct?
2. In *Example 2*, we first converted all measurements to centimetres. Could you have solved the problem without converting to centimetres? Justify your answer.
3. In *Example 3:*
   a) We multiplied the terms of one ratio by 2 and the terms of the other ratio by 3. Explain why this did not change the proportion.
   b) Could we have multiplied each ratio by numbers other than 2 and 3? If your answer is yes, give an example of these numbers.

Practice

Check

4. Find the value of each variable.
   a) $t:18 = 6:3$ b) $v:60 = 3:10$
   c) $x:15 = 2:3$ d) $s:28 = 9:4$
   e) $6:c = 2:11$ f) $39:b = 3:2$

5. Find the value of each variable.
   a) $5:t = 15:36$ b) $45:72 = 5:n$
   c) $120:70 = 12:k$ d) $81:27 = 9:m$
   e) $27:63 = p:7$ f) $8:s = 64:80$

6. Find the value of each variable.
   a) $1:6 = a:54$ b) $3:8 = e:40$
   c) $2:15 = f:75$ d) $42:36 = g:6$
   e) $3:7 = 30:p$ f) $26:65 = 2:r$

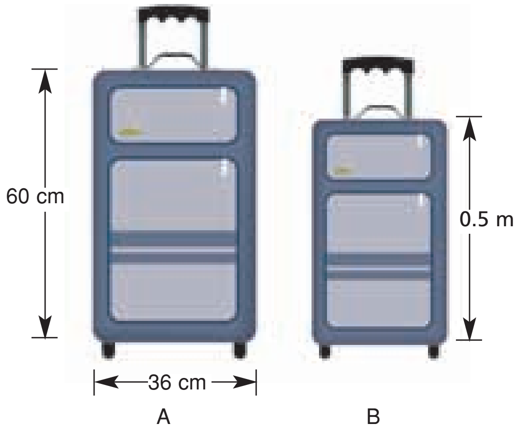
7. Find the value of each variable.
   a) $18:a = 14:21$ b) $35:b = 15:12$
   c) $m:18 = 18:27$ d) $88:33 = h:6$
   e) $6:8 = j:44$ f) $15:42 = 20:w$

8. In the NHL, the ratio of shots taken to goals scored by an all-star player is 9:2. The player has a 50-goal season. How many shots did he take?

Apply

9. An ad said that 4 out of 5 dentists recommend a certain chewing gum for their patients. Suppose 185 dentists were interviewed. Find the number of dentists who recommend this gum.

10. These suitcases have the same length-to-width ratio. Calculate the width of suitcase B.



11. **Assessment Focus** In rectangle MNPQ, the ratio of the length of MN to the length of MP is 4:5.

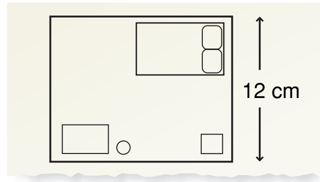
a) Does this ratio tell you how long MN is? Explain.

b) Suppose MN is 12 cm long. How long is MP? Use a diagram to illustrate your answer. Show your work.

12. The scale on a map of British Columbia is 1:5 000 000. This means that 1 cm on the map represents 5 000 000 cm actual distance. The map distance between Kelowna and Salmon Arm is 2.1 cm. What is the actual distance between these towns?

13. The scale on a map of Saskatchewan is 1 cm represents 50 km. The actual straight line distance between Regina and Saskatoon is about 257 km. What is the map distance between these 2 cities?

14. Jacob drew this picture of his bedroom.



The ratio of an actual dimension to the dimension in the drawing is 60:1.

a) The actual length of his bedroom is 9 m. What is the length of the bedroom in Jacob's drawing?

b) The width of Jacob's bedroom in the drawing is 12 cm. What is the actual width of his bedroom?

15. On a blueprint of a new house, a particular room has length 11 cm. The actual room has length 6.6 m and width 4.8 m. What is the width of the room on the blueprint? Show how you found out.

16. Fatima plants 5 tree seedlings for every 3 that Shamar plants. Shamar plants 6 trees in 1 min.

a) How many trees does Fatima plant in 1 min?

b) Did you write a proportion to solve this problem? If so, how else could you have solved it?

17. **Take It Further** Bruno wants to mix concrete to lay a walkway. The instructions call for 2 parts cement, 3 parts sand, and 4 parts gravel. Bruno has 4 m^3 of sand. How much cement and gravel should Bruno mix with this sand?

18. **Take It Further** At the movie theatre, 65 student tickets were sold for one performance.

a) The ratio of adult tickets sold to student tickets sold was 3:5. How many adult tickets were sold?

b) The ratio of adult tickets sold to child tickets sold was 3:2. How many child tickets were sold?

c) One adult ticket cost \$13. One student ticket cost \$7.50. One child ticket cost \$4.50. How much money did the performance make?

19. **Take It Further** Bella's grandfather is confined to a wheelchair. He is coming to visit her. Bella wants to build a wheelchair ramp. Her research tells her that there should be 3.5 m of ramp for every 30 cm of elevation. The distance from the ground to the front doorstep of Bella's house is 9 cm. What should the length of the ramp be?

20. **Take It Further** Forty-five students take piano lessons. The ratio of the numbers of students who take piano lessons to violin lessons is 15:8. The ratio of the numbers of students who take violin lessons to clarinet lessons is 8:9.

a) How many students take violin lessons?

b) How many students take clarinet lessons?

21. **Take It Further** Suppose you want to find the height of a flagpole.
You know your height. You can measure the length of the shadow of the flagpole.
Your friend can measure your shadow.
How can you use ratios to find the height of the flagpole? Explain.
Include a sketch in your explanation.

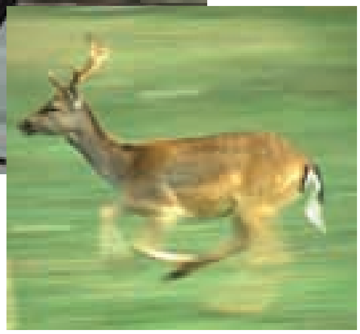
Reflect

Can you use a proportion to solve any ratio problem?
If your answer is yes, when might you not use a proportion to solve a ratio problem?
If your answer is no, give an example of a ratio problem that you *could* not use a proportion to solve.

5.9 Exploring Rates

Focus Use models and diagrams to investigate rates.

Canadian speed skater Jeremy Wotherspoon, of Red Deer, Alberta, set the world record for the 500 m at the 2004 World Cup in Italy. He skated at an average speed of 14.44 m/s. The white-tailed deer can run at speeds of up to 30 km/h.



Who is faster?
How can you find out?

Investigate

Work with a partner.

You will need a stopwatch.
Take your pulse. Your partner is the timekeeper.
Place your index and middle fingers on the side of your neck, under your jawbone.
Count the number of beats in 20 s.
Reverse roles.
Count the number of beats in 30 s.

- ➤ Who has the faster heart rate?
  How do you know?
- ➤ Estimate how many times each person's heart would beat in 1 h.
  What assumptions do you make?
  Are these assumptions reasonable?

Reflect & Share

Compare your results with those of another pair of classmates.
How can you decide who has the fastest heart rate?

Connect

When we compare two things with different units, we have a **rate**.
Here are some rates:

- We need 5 sandwiches for every 2 people.
- Oranges are on sale for $1.49 for 12.
- Gina earns $4.75 per hour for baby-sitting.
- There are 500 sheets on one roll of paper towels.

The last two rates are **unit rates**.
Each rate compares a quantity to 1 unit.

Jamal skipped rope 80 times in 1 min.
We say that Jamal's rate of skipping is 80 skips per minute.
We write this as 80 skips/min.

To find unit rates, we can use diagrams, tables, and graphs.

Example 1

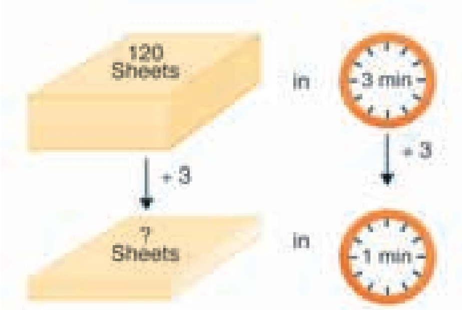
A printing press prints 120 sheets in 3 min.

a) Express the printing as a unit rate.

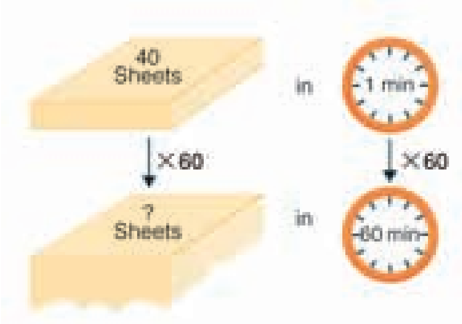
b) How many sheets are printed in 1 h?

A Solution

a) Draw a diagram.
The press prints 120 sheets in 3 min.
So, in 1 min, the press prints:
120 sheets $\div$ 3 = 40 sheets
The unit rate of printing is 40 sheets/min.



b) In 1 min, the press prints 40 sheets.
One hour is 60 min.
So, in 60 min, the press prints:
60 $\times$ 40 sheets = 2400 sheets
The press prints 2400 sheets in 1 h.



Example 2

Use the data in *Example 1*.
How long will it take to print 1000 sheets?

A Solution

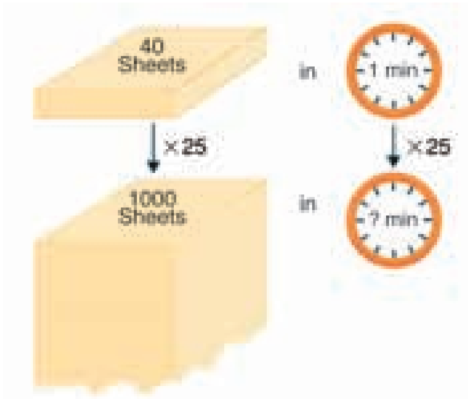
In 1 min, the press prints 40 sheets.
So, in 5 min, the press prints: $5 \times 40 = 200$ sheets
Make a table. Every 5 min, 200 more sheets are printed.
Extend the table until you get 1000 sheets.

Time (min)	5	10	15	20	25
Sheets printed	200	400	600	800	1000

The press takes 25 min to print 1000 sheets.

Example 2
Another Solution

The press prints 40 sheets in 1 min.
Think: What do we multiply
40 by to get 1000?
Use division: $1000 \div 40 = 25$
So, $40 \times 25 = 1000$
We multiply the time by the same number.
$1 \text{ min} \times 25 = 25 \text{ min}$
The press takes 25 min to print 1000 sheets.



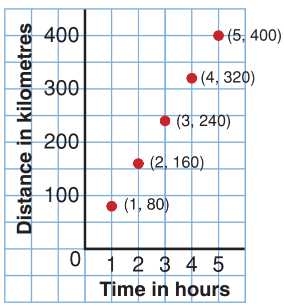
The rate at which a car travels is its average speed.
When a car travels at an average speed of 80 km/h, it travels:

80 km in 1 h
160 km in 2 h
240 km in 3 h
320 km in 4 h
400 km in 5 h … and so on

We can show this motion on a graph.
An average speed of 80 km/h is a unit rate.

At any given time, the car may be travelling at a different speed.

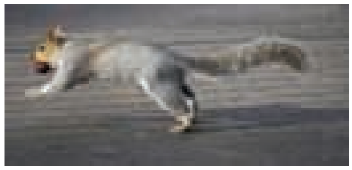
Car Travelling at 80 km/h



Example 3

a) A human walks at an average speed of 5 km/h.
What is this speed in metres per second?

b) A squirrel can run at a top speed of about 5 m/s.
What is this speed in kilometres per hour?



A Solution

a) In 1 h, a human walks 5 km.
There are 1000 m in 1 km.
So, to convert kilometres to metres, multiply by 1000.
5 km is 5×1000 m or 5000 m.
There are 60 min in 1 h.
So, in 1 min, a human walks: $\frac{5000}{60} = 83.\overline{3}$ m
There are 60 s in 1 min.
So, in 1 s, a human walks: $\frac{83.\overline{3}}{60}$ m $= 1.3\overline{8}$ m
A human walks at an average speed of about 1.4 m/s.

b) In 1 s, a squirrel can run about 5 m.
There are 60 s in 1 min.
So, in 1 min, a squirrel can run about: $5 \text{ m} \times 60 = 300$ m
There are 60 min in 1 h.
So, in 1 h, a squirrel can run about: $300 \text{ m} \times 60 = 18\,000$ m
There are 1000 m in 1 km.
So, to convert metres to kilometres, divide by 1000.
18 000 m is $\frac{18\,000}{1000}$ km or 18 km.
So, in 1 h a squirrel can run about 18 km.
A squirrel can run at a top speed of about 18 km/h.

Discuss the ideas

1. What is the difference between a rate and a unit rate?
2. How is a ratio like a rate?
3. Look at the *Examples* in *Connect*.
   a) Can a rate be expressed as a fraction? Justify your answer.
   b) Can a rate be expressed as a percent? Justify your answer.
   c) Can a rate be expressed as a decimal? Justify your answer.

Practice

Check

4. Express each unit rate using symbols.
  a) Morag typed 60 words in 1 min.
  b) Peter swam 25 m in 1 min.
  c) Abdul read 20 pages in 1 h.

5. Express as a unit rate.
  a) June cycled 30 km in 2 h.
  b) A caribou travelled 12 km in 30 min.
  c) A plane flew 150 km in 15 min.

6. Express as a unit rate.
  a) Elsie delivered 220 flyers in 4 h.
  b) Winston iced 90 cupcakes in 1.5 h.
  c) The temperature rose 18°C in 4 h.

7. Which sentences use ratios? Which sentences use rates? How do you know?
  a) The car was travelling at 60 km/h.
  b) Katie earns $8/h at her part-time job.
  c) The punch has 3 cups of cranberry juice and 4 cups of ginger ale.
  d) The soccer team won 25 games and lost 15 games.

Apply

8. Find the unit price for each item.
  a) Milk costs $4.50 for 4 L.
  b) Corn costs $3.00 for 12 cobs.
  c) 24 cans of iced tea cost $9.99.

9. Lindsay has scored 15 goals in 10 lacrosse games this season.
  a) What is her unit rate of scoring?
  b) How many goals might Lindsay score in 35 games? What assumptions do you make?

10. Before running in a 100-m race, Gaalen's heart rate was 70 beats/min. Which do you think is more likely after the race: 60 beats/min or 120 beats/min? Explain.

11. Ribbon costs $1.44 for 3 m.
  a) What is the cost per metre?
  b) How much would 5 m of ribbon cost?
  c) How much ribbon could you buy for $12?

12. Monique worked as a cleaner at the Calgary Stampede and Exhibition. She was paid $84 for an 8-h day.
  a) What was Monique's hourly rate of pay?
  b) How much would Monique earn for 35 h of work?

13. A 400-g package of Swiss cheese costs $4.80.
  a) What is the cost per 100 g?
  b) How much would 250 g cost?
  c) How much would 1 kg cost?
  d) How much Swiss cheese could you buy with $18?

14. Write each speed in metres per second.

a) The sailfish is the fastest swimmer, reaching a speed of up to 109 km/h.

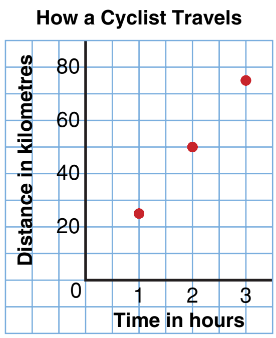
b) A bald eagle can fly at a top speed of about 50 km/h.

15. Write each speed in kilometres per hour.

a) A pitcher throws a baseball. The ball crosses home plate at a speed of 40 m/s.

b) A cockroach can travel at a speed of 1.5 m/s.

16. The graph shows how a cyclist travelled in 3 h.



a) How far did the cyclist travel in 1 h?

b) What is the average speed of the cyclist? How do you know?

17. James and Lucinda came from England to Canada on holiday. The rate of exchange for their money was \$2.50 Can to £1.

a) How many Canadian dollars would James get for £20?

b) What is the value in English pounds of a gift Lucinda bought for \$30 Can?

18. **Assessment Focus** Petra works on an assembly line. She can paint the eyes on 225 dolls in 1 h.

a) How many dolls can she paint in 15 min?

b) How many dolls can she paint in 30 s?

What assumptions do you make?

19. **Take It Further** When a person runs a long-distance race, she thinks of the time she takes to run 1 km (min/km), rather than the distance run in 1 min (km/min). On a training run, Judy took 3 h 20 min to run 25 km. What was Judy's rate in minutes per kilometre?

20. **Take It Further** Leo trained for the marathon. On Day 1, he took 70 min to run 10 km. On Day 10, he took 2 h 40 min to run 20 km. On Day 20, he took 4 h 15 min to run 30 km.

a) What was Leo's running rate, in minutes per kilometre, for each day?

i) Day 1 **ii)** Day 10 **iii)** Day 20

b) What do you think Leo's running rate, in minutes per kilometre, might be for the 44 km of the marathon? How long do you think it will take him? Why do you think so?

Reflect

Explain how ratios and rates are different.
Describe a situation where you might use a ratio.
Describe a situation where you might use a rate.

5.10 Comparing Rates

Focus Use unit rates to compare rates.

Many grocery items come in different sized packages.

250 mL	500 mL	1 L	2 L
\$1.49	\$1.69	\$2.79	\$3.99

How can you find out which is the best buy?

Investigate

Work on your own.

Use a calculator if it helps.

12 garbage bags for \$1.99

48 garbage bags for \$5.29

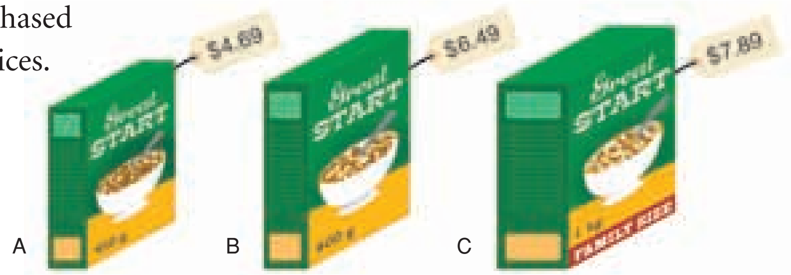
Which box of garbage bags is the better buy?
What do you need to consider before you decide?

Reflect & Share

Compare your results with those of a classmate.
Did both of you choose the same box? If so, justify your choice.
If not, can both of you be correct? Explain.

Connect

Great Start cereal can be purchased in three different sizes and prices.



The smallest box costs the least, but that does not mean it is the best buy.
Find the unit cost for each box of cereal, then compare the unit costs.
It is difficult to calculate the cost of 1 g; so, we calculate the cost of 100 g for each box of cereal.

Box A has mass 450 g and costs \$4.69.
450 g is 4.5×100 g; so, the cost of 100 g of Box A is:
$\frac{\$4.69}{4.5} \doteq \$1.04$

Box B has mass 600 g and costs \$6.49.
The cost of 100 g of Box B is: $\frac{\$6.49}{6} \doteq \$1.08$

Box C has mass 1000 g and costs \$7.89.
The cost of 100 g of Box C is: $\frac{\$7.89}{10} \doteq \$0.79$

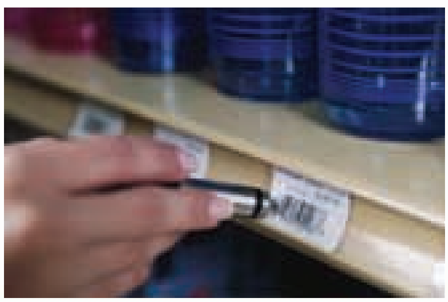
Each unit cost can be written as a unit rate.
The “unit” for unit rate is 100 g.
The least unit rate is \$0.79/100 g, for Box C.
The greatest unit rate is \$1.08/100 g, for Box B.

Math Link

Your World

In a supermarket, the tag on the shelf with the bar code of an item often shows the cost of 1 g of the item.

With this information, you can compare the costs of packages of the item in different sizes.



Example 1

Paper towels can be purchased in 3 different sizes and prices.

A	B	C
2 rolls $0.99	6 rolls $3.99	12 rolls $4.99

Which package is the best buy?
How do you know?

A Solution

Find the unit cost of each package, then compare the unit costs.
Since 12 is a multiple of both 2 and 6, use 12 rolls as the unit.

Package A:
12 rolls is 6×2 rolls; so, the cost of 12 rolls is:
$\$0.99 \times 6 = \$5.94$

Package B:
12 rolls is 2×6 rolls; so, the cost of 12 rolls is:
$\$3.99 \times 2 = \$7.98$

Package C:
The cost of 12 rolls is $4.99.

The least unit rate is $4.99/12 rolls, for Package C.
So, the best buy is Package C.

Example 2

Mariah is looking for a part-time job. She wants to work 15 h a week.
She has been offered three positions.

Day Camp Counsellor	Cashier	Library Assistant
$7.50 per hour	$25.00 for 3 h	$44.00 for 5 h

a) Which job pays the most?
b) For the job in part a, how much will Mariah earn in one week?

A Solution

a) Calculate the unit rate for each job.
The unit rate is the hourly rate of pay.
For day camp counsellor, the unit rate is \$7.50/h.
For cashier, the unit rate is: $\frac{\$25.00}{3\text{ h}} \doteq \$8.33/\text{h}$
For library assistant, the unit rate is: $\frac{\$44.00}{5\text{ h}} = \$8.80/\text{h}$
The library assistant job pays the most.

b) Mariah works 15 h a week, at a rate of \$8.80/h.
She will earn: $15 \times \$8.80 = \$132.00$
Mariah will earn \$132.00 a week as a part-time library assistant.

Discuss the ideas

1. In *Connect*, we compared the prices of cereal by dividing to find unit rates, where 100 g was the unit.
   a) What other unit could we have used for the unit rate?
   b) What is the price for each cereal at the unit rate in part a?

2. In *Connect*, the best buy for the cereal is Box C. Why might you not buy Box C?

3. In *Connect*, why did we not use a unit of 1 kg for the cereal?

4. For the cereal, we used a unit of 100 g. For the paper towels, we used a unit of 12 rolls. Which strategy did you find easier to understand? Justify your choice.

Practice

Check

5. Write a unit rate for each statement.
   a) \$399 earned in 3 weeks
   b) 680 km travelled in 8 h
   c) 12 bottles of juice for \$3.49
   d) 3 cans of soup for \$0.99

6. Which is the greater rate? How do you know?
   a) \$24.00 in 3 h or \$36.00 in 4 h
   b) \$4.50 for 6 muffins or \$6.00 for 1 dozen muffins
   c) \$0.99 for 250 mL or \$3.59 for 1 L

7. Delaney goes to the store to buy some mushroom soup. She finds that a 110-mL can costs $1.49. A 500-mL can of the same brand costs $4.29.

a) Which is the better buy?

b) Delaney buys the 110-mL can. Why might she have done this?

c) Another customer bought a 500-mL can. How might you explain this?

Apply

8. Which is the better buy?

a) 5 grapefruit for $1.99 or 8 grapefruit for $2.99

b) 2 L of juice for $4.49 or 1 L of juice for $2.89

c) 100 mL of toothpaste for $1.79 or 150 mL of toothpaste for $2.19

d) 500 g of yogurt for $3.49 or 125 g of yogurt for $0.79

9. Mr. Gomez travelled 525 km in 6 h.

a) Assume Mr. Gomez travelled the same distance each hour. What is this distance?

b) How is the distance in part a related to the average speed?

c) At this rate, how long will it take Mr. Gomez to travel 700 km?

10. **a)** Which is the greatest average speed?

i) 60 km in 3 h

ii) 68 km in 4 h

iii) 70 km in 5 h

b) Draw a graph to illustrate your answers in part a.

11. Each week, Petra earns $370 for 40 h of work as a lifeguard. Giorgos earns $315 for 35 h of work as a starter at the golf course.

a) Which job pays more?

b) Would you take the job in part a instead of the other job? Justify your answer.

12. In the first 9 basketball games of the season, Lashonda scored 114 points.

a) On average, how many points does Lashonda score per game?

b) At this rate, how many points will Lashonda score after 24 games?

13. Lakelse Lake, BC, had the most snow for one day in Canada, which was 118.1 cm. Assume the snow fell at a constant rate. How much snow fell in 1 h?

14. **Assessment Focus** Becky's dog will eat two different brands of dog food. Brand A costs \$12.99 for a 3.6-kg bag. Brand B costs \$39.99 for an 18.1-kg bag.

a) Without calculating, which brand do you think is the better buy? Explain your choice.

b) Find the unit cost of each brand of dog food.

c) Which brand is the better buy? How does this compare with your prediction in part a?

d) Why might Becky not buy the brand in part c?

15. The food we eat provides energy in calories. When we exercise, we burn calories. The tables show data for different foods and different exercises.

Food	Energy Provided (Calories)
Medium apple	60
Slice of white bread	70
Medium peach	50
Vanilla fudge ice cream	290
Chocolate iced doughnut	204

Activity	Calories Burned per Hour
Skipping	492
Swimming	570
Cycling	216
Aerobics	480
Walking	270

These data are for a female with a mass of 56 kg. The data vary for men and women, and for different masses.

a) How long would a person have to:

i) cycle to burn the calories after eating one apple?

ii) walk to burn the calories after eating two slices of bread?

b) Suppose a person ate vanilla fudge ice cream and a chocolate iced doughnut.

i) Which activity would burn the calories quickest? How long would it take?

ii) Which 2 activities would burn the calories in about 2 h?

c) Use the data in the tables to write your own problem. Solve your problem. Show your work.

16. A 2.5-kg bag of grass seed covers an area of 1200 m^2. How much seed is needed to cover a square park with side length 500 m?

17. Suppose you were asked to tutor another student.

a) How would you explain $\frac{40}{5}$ as a rate? What real-life example could you use to help?

b) How would you explain $\frac{1.75}{100}$ as a unit rate? What real-life example could you use to help?

18. **Take It Further** This table shows the city fuel consumption for 5 different cars.

Car	Fuel Consumption
Toyota Echo	26.8 L/400 km
Ford Focus	23.0 L/250 km
Honda Civic	11.25 L/150 km
Saturn Ion	33.25 L/350 km
Hyundai Accent	16.2 L/200 km

a) Which of these cars is the most economical for city driving?

b) Over a distance of 500 km, how much more fuel is needed to drive the least fuel-efficient car compared to the most fuel-efficient car? How did you find out? What assumptions do you make?

19. **Take It Further** A local garden nursery sells a 2-kg bag of plant food for $6.99. Hayley decides to market her own plant food. She is going to sell it in a 2.5-kg bag. What is the most Hayley should sell her plant food for to undercut the competitor? How did you find out?

20. **Take It Further** Population density is expressed as a rate. It compares the number of people in a population with the area of the land where they live. Population density is measured in number of people per square kilometre.

a) Find the population density for each country.

i) Canada: 32 623 490 people in 9 984 670 km^2

ii) China: 1 307 560 000 people in 9 562 000 km^2

iii) Japan: 127 760 000 people in 377 727 km^2

b) How do the population densities in part a compare?

21. Troy rides his bike to school. He cycles at an average speed of 20 km/h.
It takes Troy 24 min to get to school.

a) How far is it from Troy's home to school?

b) One morning, Troy is late leaving. He has 15 min to get to school. How much faster will Troy have to cycle to get to school on time? Explain.

Reflect

What is a unit rate?
Describe the types of problems you can solve using unit rates.
Write your own problem that involves a unit rate.

Unit Review

What Do I Need to Know?

- ☑ To calculate a *percent decrease*: divide the decrease by the original amount, then write the quotient as a percent.
  Percent decrease (%) $= \frac{\text{Decrease}}{\text{Original amount}} \times 100$

- ☑ To calculate a *percent increase*: divide the increase by the original amount, then write the quotient as a percent.
  Percent increase (%) $= \frac{\text{Increase}}{\text{Original amount}} \times 100$

- ☑ A *part-to-whole ratio* can be written in fraction form and as a percent.
  For example:
  There are 9 girls in a class of 20 students.
  The ratio of girls to all the students is: 9:20, or $\frac{9}{20}$
  This can be written as a percent: $\frac{9}{20} = \frac{45}{100} = 45\%$
  So, 45% of the students are girls.

- ☑ An *equivalent ratio* can be formed by multiplying or dividing the terms of a ratio by the same number.
  For example,
  $10:16 = (10 \times 2):(16 \times 2)$
  $= 20:32$
  $10:16 = \frac{10}{2}:\frac{16}{2}$
  $= 5:8$
  5:8, 10:16, and 20:32 are equivalent ratios.

- ☑ A *proportion* is a statement that two ratios are equal.
  For example, $x:3 = 6:9$

- ☑ A *rate* is a comparison of two quantities with different units.
  For example, 500 km in 4 h is a rate.
  Divide 500 km by 4 h to get the *unit rate* of $\frac{500 \text{ km}}{4}$, or 125 km/h.

What Should I Be Able to Do?

LESSON

5.1

1. Write each decimal as a fraction and as a percent.

a) 0.65 **b)** 0.0069

c) 0.0375 **d)** 0.9825

2. Conner got 21 out of 24 on a science quiz. Rose got $83.\overline{3}\%$ on the quiz. Who did better? How did you find out?

3. Write each percent as a fraction and as a decimal.

a) 38% **b)** 93.75% **c)** 0.79% **d)** 0.2%

5.2

4. Write each percent as a decimal.

a) 160% **b)** 310% **c)** 0.27% **d)** 0.9%

5. The average attendance at a regular season home game of the Winnipeg Blue Bombers in 2006 was 26 988. The attendance at the 2006 Grey Cup game in Winnipeg was about 166% of the regular season attendance.

a) How many people attended the 2006 Grey Cup game?

b) Estimate to check your answer is reasonable.

5.3

6. Joline collects hockey cards. She needs 5 cards to complete a set. This is 20% of the set. How many cards are in the set? Justify your answer.

7. Wei's mark on a test was 39. This mark was 60%. What was the total possible mark?

8. Two mandrills, Amy and Joe, were born at the zoo. Each mandrill had a birth mass of 1 kg. After Month 1, Amy's mass was 25% greater than her birth mass. After Month 2, Amy's mass was 20% greater than her mass after Month 1. After Month 1, Joe's mass was 20% greater than his birth mass. After Month 2, Joe's mass was 25% greater than his mass after Month 1.

a) Which mandrill had the greater mass after Month 2? Explain.

b) Could you have found Amy's mass after Month 2 by finding 145% of her birth mass? Justify your answer.

LESSON

9. A ball bounced 64% of the height from which it was dropped. The bounce was 72 cm high. What is the height from which the ball was dropped?

10. A new queen size bed sheet measures 210 cm by 240 cm. The length and width each shrinks by 2% after the first wash.
    a) What are the dimensions of the sheet after washing?
    b) What is the percent decrease in the area of the sheet?

5.4

11. A portable CD player is on sale. It is advertised as, "Save $20. You pay $49.99."
    a) What is the regular price?
    b) What is the percent decrease?

12. A gym suit sells for $89.99 at Aerobics for All in Victoria, BC. In August, there is a 25% discount as a *Back to School* special. Calculate the total price including taxes.

13. The regular price of a sleeping bag is $39.95. It is on sale for 20% off. There is a 13% sales tax. The discount is usually applied before the tax is added. Suppose the tax is calculated first. Would the total cost be more or less in this case? Explain.

5.5

14. A box contains 5 cream chocolates, 2 chocolate-covered almonds, and 3 caramel chocolates.
    a) What is each ratio below? Sketch a picture for each ratio. When possible, write the ratio as a percent.
        i) almond chocolates to caramel chocolates
        ii) cream chocolates to caramel chocolates
        iii) cream chocolates to all chocolates
        iv) cream chocolates to almond chocolates to caramel chocolates
    b) Lesley ate one of each kind of chocolate. What is each new ratio for part a?

15. In Mary's closet, there are 7 T-shirts, 4 pairs of shorts, and 3 sweatshirts. Write each ratio in as many different ways as you can.
    a) T-shirts to shorts
    b) sweatshirts to shorts
    c) sweatshirts to T-shirts and shorts

16. a) Draw two different diagrams to show the ratio 5:6.
    b) Draw a diagram to show the ratio 5:3.
    c) Draw a diagram to show the ratio 4:1:2.

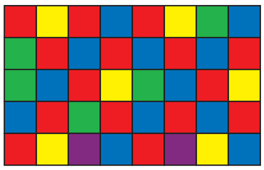
LESSON 5.6

17. a) Write each ratio below in simplest form.

  i) green squares to red squares
  ii) yellow squares to purple squares
  iii) red squares to total number of squares
  iv) purple squares to blue squares to yellow squares

b) State the colours for each ratio.

  i) 1:6 **ii)** 2:5 **iii)** 2:4:5



18. In Ms. Bell's class, the ratio of boys to girls is 5:4.

a) There are 15 boys in the class. How many girls are there?

b) Two girls leave. What is the new ratio of boys to girls?

19. Explain two different ways to get ratios equivalent to 25:10:30.

20. Find the value of the variable in each pair of equivalent ratios.

a) 3:7 and h:70

b) 3:8 and 15:r

c) s:42 and 5:6

d) 84:t:96 and 21:27:24

5.7

21. Write each ratio with second term 1.

a) 32:4 **b)** 132:144

c) 9:6 **d)** 22:8

22. A jug of orange juice requires 3 cans of orange concentrate and 5 cans of water.

a) Accidentally, 4 cans of concentrate were mixed with 5 cans of water. Is the mixture stronger or weaker than it should be? Explain.

b) Suppose 6 cans of water were mixed with 3 cans of concentrate. Is the mixture stronger or weaker than it should be? Explain.

23. The ratio of computers to students in Ms. Beveridge's class is 2:3. The ratio of computers to students in Mr. Walker's class is 3:5. Each class has the same number of students.

a) Which room has more computers? How did you find out?

b) Did you use percent to find out? Why or why not?

5.8

24. The numbers of bass and pike in a lake are estimated to be in the ratio of 5:3. There are approximately 300 bass in the lake. About how many pike are there?

25. The St. Croix junior hockey team won 3 out of every 4 games it played. The team played 56 games. How many games did it lose?

LESSON

26. A punch recipe calls for orange juice and pop in the ratio of 2:5. The recipe requires 1 L of pop to serve 7 people.
   a) How much orange juice is needed for 7 people?
   b) About how much orange juice and pop do you need to serve 15 people? 20 people? Justify your answers.

5.9

27. Express as a unit rate.
   a) A bus travelled 120 km in 3 h.
   b) An athlete ran 1500 m in 6 min.
   c) A security guard earned $16.00 for 2 h of work.

28. A cougar can run 312 m in 20 s. A wild horse can run 200 m in 15 s.
   a) Which animal is faster?
   b) What is the ratio of their average speeds?

29. Write each speed in metres per second.
   a) The dolphin can swim at a top speed of 60 km/h.
   b) The polar bear can run at a top speed of 40 km/h.

30. Milena worked in the gift shop of the Costume Museum of Canada in downtown Winnipeg. She was paid $57 for a 6-h shift.
   a) What was Milena's hourly rate of pay?
   b) How much would Milena earn for 25 h of work?

5.10

31. a) Find the unit cost of each item.
      i) 4 L of milk for $4.29
      ii) 2.4 kg of beef for $10.72
      iii) 454 g of margarine for $1.99
   b) For each item in part a, which unit did you choose? Justify your choice.

32. Which is the better buy? Justify each answer.
   a) 6.2 L of gas for $5.39 or 8.5 L of gas for $7.31
   b) 5 candles for $3.00 or 12 candles for $5.99
   c) 2 kg of grass seed for $1.38 or 5 kg of grass seed for $2.79

33. Kheran ran 8 laps of the track in 18 min. Jevon ran 6 laps of the track in 10 min. Who had the greater average speed? How do you know?

34. Each week, Aaron earns $186 for 24 h of work as a ticket seller. Kayla earns $225 for 30 h of work as a cashier. Which job pays more? How did you find out?

Practice Test

1. Find.

a) 165% of 80 **b)** $3\frac{1}{2}$% of 400

c) 0.4% of 500 **d)** 35% of 51

2. Find the value of the variable in each proportion.

a) $3:5 = g:65$

b) $h:8 = 56:64$

c) $16:m = 20:35$

d) $36:8 = 81:f$

3. Express as a unit rate.

a) Ethan earned $41.25 in 8 h.

b) Brianna completed 8 Sudoku puzzles in 96 min.

c) A car travelled 20 km in 15 min.

4. The Grade 8 class at Wheatly Middle School sold 77 boxes of greeting cards to raise money for the under privileged. This is 22% of all the boxes of greeting cards sold. Calculate how many boxes were sold.

5. The manager of a clothing store reduces the price of an item by 25% if it has been on the rack for 4 weeks.
The manager reduces the price a further 15% if the item has not been sold after 6 weeks.

a) What is the sale price of the jacket after 6 weeks?

b) Calculate the sale price including PST of 5% and GST of 6%.

6. The price of a house increased by 20% from 2004 to 2005.
The price of the house decreased by 20% from 2005 to 2006.
Will the price of the house at the beginning of 2004 be equal to its price at the end of 2006?
If your answer is yes, show how you know.
If your answer is no, when is the house less expensive?

7. Which is the better buy?

a) 8 batteries for \$3.49 or 24 batteries for \$9.29

b) 100 g of iced tea mix for \$0.29 or 500 g of iced tea mix for \$1.69
Show your work.

8. In the baseball league, the Leos play 8 games.
Their win to loss ratio is 5:3.
The Tigers play 11 games.
Their win to loss ratio is 7:4.

a) Which team has the better record?

b) Suppose the Leos win their next game and the Tigers lose theirs.
Which team would have the better record? Explain.

9. Taylor is comparing the prices of 3 different sizes of Rise and Shine orange juice.

1 L for \$2.79   1.84 L for \$3.99   2.78 L for \$6.29

Should Taylor buy the largest carton?
Justify your answer.

10. Use the number $\frac{3}{4}$.
Provide a context to explain each meaning of $\frac{3}{4}$.

a) as a fraction   **b)** as a ratio

c) as a quotient   **d)** as a rate

Unit Problem What Is the Smartest, Fastest, Oldest?

Mr. Peabody has three **hypotheses** about the animal world. For a Science Fair project, you decide to analyse data to test each of Mr. Peabody's hypotheses.

A **hypothesis** is something that seems likely to be true. It needs to be tested to be proved or disproved.

Part A

Mr. Peabody believes you can predict an animal's intelligence by looking at the size of its brain. He thinks the smartest animal has the greatest brain mass compared with its body mass. Investigate this hypothesis.

Species	Comparing Mass	
	Body Mass (g)	Brain Mass (g)
Cat	3 300	30
Monkey	7 000	100
Human	56 000	1400

1. Find each ratio of brain mass to body mass.
- How does a human compare to a cat?
- How does a human compare to a monkey?
- How does a cat compare to a monkey?

According to Mr. Peabody's hypothesis, which animal is smartest? Explain your reasoning.

Part B

Mr. Peabody believes that among animals of the same family, the lesser the average mass of a species, the greater the average running speed. Here are some data for the cat family.

Species	Average Mass (kg)	Running Rate
Cheetah	45	150 m in 5 s
Lion	250	300 m in 15 s
Leopard	50	170 m in 10 s

2. Find the average speed of each animal in metres per second, metres per minute, metres per hour, and kilometres per hour. Do the data support Mr. Peabody's hypothesis that the lesser average mass indicates a greater average speed? Explain your reasoning.

Part C

Mr. Peabody believes that an animal's heart rate can be used to predict its life expectancy. The animal with the least heart rate will have the greatest life expectancy.

Species	Heart Rate	Average Life Span
Cat	30 beats per 12 s	15 years
Monkey	48 beats per 15 s	25 years
Human	10 beats per 10 s	70 years

Check List

Your work should show:

- ✓ how you calculated each ratio and rate
- ✓ your conclusions and reasoning for each hypothesis
- ✓ the correct use of math language

3. Find the heart rate of each animal in beats per minute.
Which animal has the greatest heart rate? The least heart rate?
Calculate the number of heartbeats in the average life span of each animal. What assumptions did you make?
Do the data support Mr. Peabody's hypothesis that a lesser heart rate indicates a longer life expectancy?
Explain your reasoning.

Part D

Review your results. Write a short letter to Mr. Peabody telling him whether you agree or disagree with each of his hypotheses, and explain why. Use math language to support your opinions.

Reflect on Your Learning

How are ratio, rate, and percent related?
How can you use your knowledge of ratios, rates, and percents outside the classroom?
Include examples in your explanation.