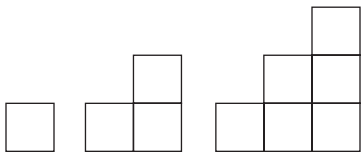
UNIT 6

Equations

Just for Fun

Pharaoh's Staircase



You need 6 blocks to make a *Pharaoh's Staircase* with a height of 3 blocks.

You will need _____ blocks for a staircase with a height of 5 blocks.

You will need _____ blocks for a staircase with a height of 10 blocks.

Pairing Up

Find the missing numbers.

6	3	5	1
4		5	9

5	8	7	11
3	6		9

3	5	8	
6	10		20

Heading Home

A Game for 2

Use the dot grid. Start at the centre, O.

You may move up, down, left, or right, but not diagonally.

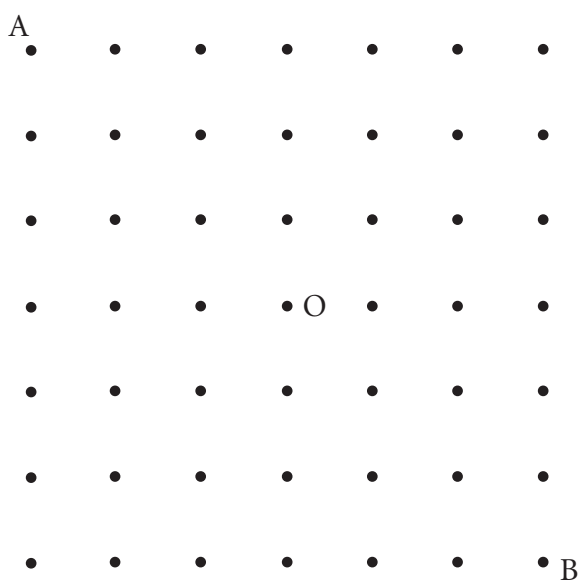
Player A draws an arrow 1 grid unit long in any direction from O.

Player B adds an arrow 1 grid unit long to Player A's arrow to make a continuous route. Take turns drawing arrows.

Player A tries to head home to A, while player B tries to head home to B.

You may not go over a dot more than once.

The first player to get home wins.



Activating Prior Knowledge

Writing Expressions and Equations

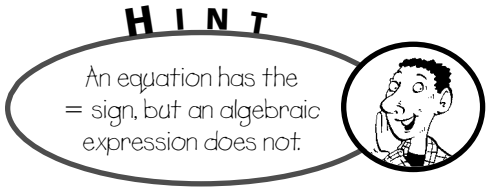
You can use algebraic expressions to represent word statements.
In an algebraic expression, a letter, such as x or n, is used to represent a number.
This letter is called a **variable**.
Multiplication of a number and a letter is written as the number followed by the letter.
For example, $4n$ means $4 \times n$.

An equation is a statement that two algebraic expressions are equal. One of them can be a number. For example, $4n = 8$ is an equation and $4n + 2 = 10$ is also an equation.

Example 1

a) Write an algebraic expression for this statement:
Five times a number minus 3

b) Write an equation for this sentence:
Three less than five times a number is 2.



Solution

Let x be the number.

a) Then, five times a number is $5x$.
$5x$ minus 3 is: $5x - 3$
$5x - 3$ is an algebraic expression.

b) Three less than five times the number is 2.
So, $5x - 3 = 2$
$5x - 3 = 2$ is an equation.

Check

1. Write an algebraic expression for each statement.

a) eight more than a number _____

b) two less than seven times a number _____

c) a number divided by 6 _____

2. Write an equation for each sentence.

a) The sum of 10 and a number is 15. _____

b) The product of a number and nine is 63._____

c) Eleven decreased by 2 times a number is 1. _____

Evaluating Expressions

To find the value of an expression, replace each variable with its given value. Then use the order of operations to simplify.

Example 2

Evaluate the expression $a + 2$ for $a = -3$.

Solution

Substitute $a = -3$ into the expression.

$a + 2 = -3 + 2$

$= -1$

Check

3. Evaluate each expression.

a) $x - 9$ for $x = -5$

$x - 9 =$ ___ $- 9$

$=$ _____

b) $4x$ for $x = 3$

$4x = 4($ $)$

$=$ _____

c) $-6 + x$ for $x = -2$

$-6 + x = -6 +$ ( )

$=$ _____

4. Evaluate each expression for $m = -3$.

a) $m - 2$

$=$ ____ $- 2$

$=$ ____

b) $m + 2$

$=$ ___ $+ 2$

$=$ _____

c) $2 - m$

$= 2 -$ _____

$=$ _____

$=$ _____

5. Evaluate each expression.

a) $a - 4$, when $a = -3$: _____

b) $-a + 3$, when $a = -4$: _____

c) $-a - 3$, when $a = -3$: _____

d) $-a - 4$, when $a = -4$: _____

e) $-a + 4$, when $a = -3$: _____

6.1 Solving Equations

Quick Review

At Home At School

➤ When you **solve an equation** you find the value of the variable that makes the equation true.

You can solve an equation by *systematic trial* or by *inspection.*

➤ Sharon baby-sits for an hourly wage.
She works for 2 h and is given an extra $3 as a tip.
Sharon earns $17. What is her hourly rate?

Let d dollars per hour represent Sharon's hourly rate.

Then $2 \times d$, or $2d$ is how much she earns for 2 h work.
Include the $3 tip, then an equation that represents this situation is: $2d + 3 = 17$

Tip

Each trial provides information to guide you in choosing a value for the next trial.

Solve by Systematic Trial

$2d + 3 = 17$

Choose a value for d and substitute.

Try $d = 10$; then $2 \times 10 + 3 = 23$	23 is too large. Try a lesser value.
Try $d = 5$; then $2 \times 5 + 3 = 13$	13 is too small. Try a value between 5 and 10.
Try $d = 6$; then $2 \times 6 + 3 = 15$	15 is too small. Try a value between 6 and 10.
Try $d = 7$; then $2 \times 7 + 3 = 17$	$d = 7$ makes the equation true.

So, $d = 7$

Solve by Inspection

$2d + 3 = 17$

To solve the equation by inspection, ask yourself:
"Which number added to 3 gives 17?"

$2d + 3 = 17$
You know that $14 + 3 = 17$
So, $2d = 14$

Then ask yourself, "Two times which number gives 14?"
You know that $2 \times 7 = 14$.
So, $d = 7$
Sharon earns $7/h.

Practice

1. Look at these algebraic expressions and equations.

$2p = 16$ $x + 12$

$\frac{n}{5} = 4$ $z - 6 = 20$

$\frac{k+3}{2}$

a) Which are expressions? _____

b) Which are equations? _____

2. Solve by inspection.

a) $2n = 12$ _____

b) $x + 10 = 30$ _____

c) $25 - p = 20$ _____

d) $x - 8 = 2$ _____

e) $5n = 15$ _____

f) $\frac{x}{2} = 5$ _____

3. Solve the equation $2x + 5 = 37$ by systematic trial.

Sample Answer:

Input	Evaluate $2x + 5$	Too large or too small?
$x = 30$	$2 \times 30 + 5 = 65$	This is too large.
$x = 20$	$2 \times 20 + 5 = 45$	This is too large.
$x = 5$	$2 \times 5 + 5 = 15$	This is too small.
$x = 10$	$2 \times 10 + 5 =$	
$x = 15$	$2 \times 15 + 5 =$	

4. Solve.

a) $3x = 60$ _____

b) $x + 12 = 30$ _____

c) $\frac{x}{5} = 9$ _____

d) $5x - 4 = 26$ _____

5. Which value of n makes the equation $\frac{20}{n} + 5 = 9$ true? Circle your answer.

a) $n = 1$ **b)** $n = 2$ **c)** $n = 4$ **d)** $n = 5$ **e)** $n = 10$ **f)** $n = 20$

6. Jasmine has 135 butterfly stickers.
She gave 15 to her little sister and the rest to her friends.
Each friend has 20 stickers. How many friends did she give stickers to?
Fill in the missing expressions to create an equation you can solve.
Let f represent the number of friends.

_____ + _____ = _____

number of stickers to sister	number of stickers to friends	total number of stickers

Jasmine gave stickers to _____ friends.

7. Write an equation you could use to solve each problem.
Solve each equation.

a) Joshua purchased tennis balls for $8 each. He spent $128.
How many tennis balls did Joshua buy?

The equation is: _____

Joshua bought ____ tennis balls.

b) Five more than three times a number is 35.
What is the number?

The equation is: _____

The number is: ____

c) A box of apples is divided among 6 people.
Each person received 3 apples.
How many apples were in the box?

The equation is: _____

There were _____ apples in the box.

d) Petra works for 5 h. She gets a bonus of $10.
Petra's total earnings are $70. What is her hourly rate?

The equation is: _____

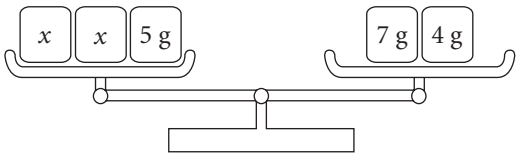
Petra's hourly rate is _____.

6.2 Using a Model to Solve Equations

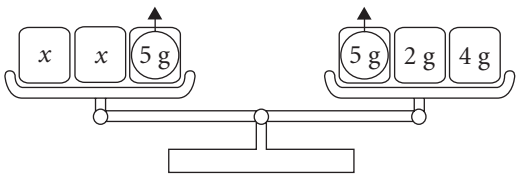
Quick Review

At Home At School

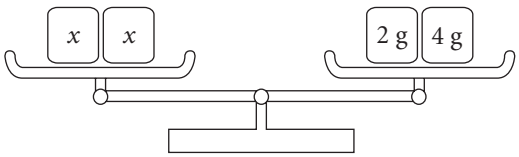
Balance scales can be used to model an equation.
When the pans are balanced, the masses on both pans are equal.
This two-pan balance models the equation $2x + 5 = 7 + 4$.



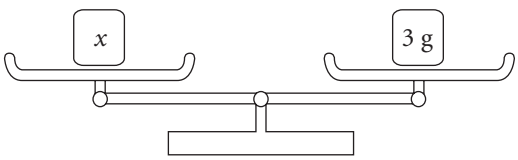
To find each unknown mass, x, replace the 7 g in the right pan with 5 g and 2 g.
Then remove 5 g from each pan.



The unknown masses have been isolated in the left pan, and 6 g is left in the right pan.



The two unknown masses balance 6 g.
So, each unknown mass is 3 g.

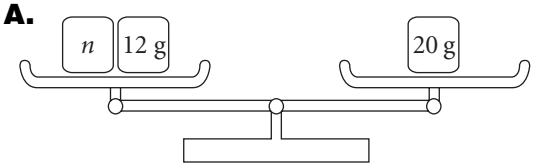


The solution to the equation is $x = 3$.
You can verify the solution by replacing each unknown mass in the original balance scales with 3 g.
Then, in the left pan: 3 g + 3 g + 5 g = 11 g
And, in the right pan: 7 g + 4 g = 11 g
Since the masses are equal, the solution is correct.

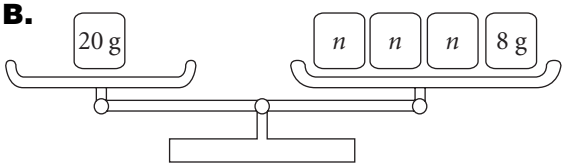
Practice

1. Match each balance scales with an equation below.

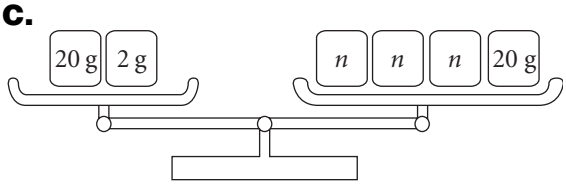
A.



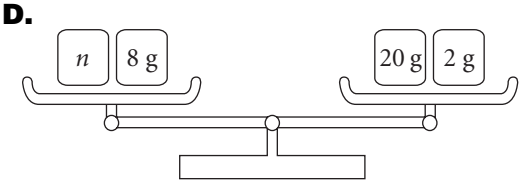
B.



C.



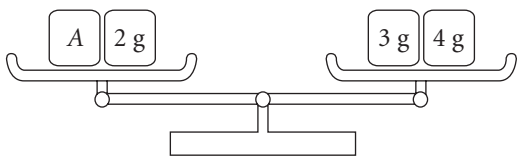
D.



i) $n + 8 = 22$ ____ **ii)** $3n + 8 = 20$ ____ **iii)** $3n + 20 = 22$ ____ **iv)** $n + 12 = 20$ ____

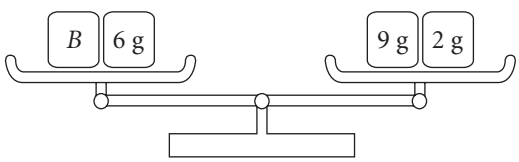
2 . Write the equation modelled by each two-pan balance. Then solve the equation.

a)



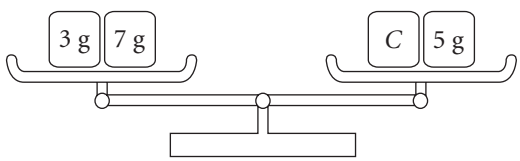
_____

b)



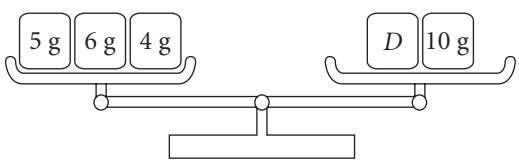
_____

c)



_____

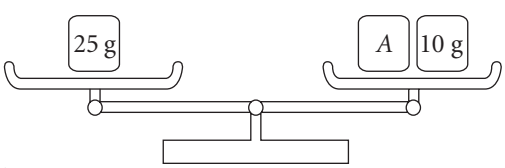
d)



_____

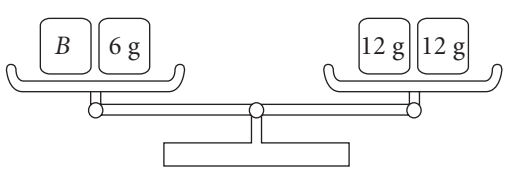
3. Find the value of the unknown mass on each balance scales.

a)



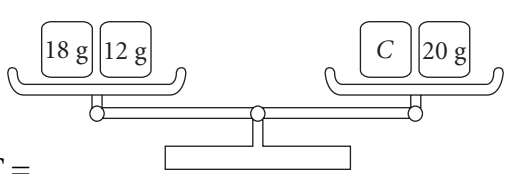
$A =$ ____

b)



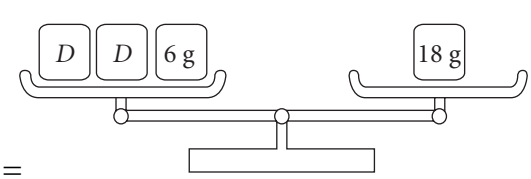
$B =$ ____

c)



$C =$ ____

d)



$D =$ ____

4. **a)** Sketch balance scales to represent each equation.

b) Solve each equation. Verify the solution.

i) $x + 7 = 12$

$x =$ _____

ii) $n + 18 = 22$

$n =$ ___

iii) $2m = 26$

$m =$ _____

iv) $27 = 9 + 3k$

$k =$ _____

5. **a)** Write an equation for each sentence.

b) Solve each equation. Verify the solution.

i) Two more than a number is 12.

_____

ii) A number increased by nine is 21.

_____

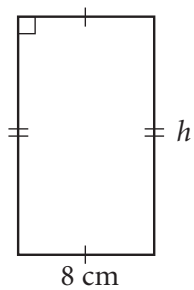
iii) Four times a number is 24.

_____

iv) Four more than three times a number is 28.

_____

6. The perimeter of this rectangle is 44 cm and the base is 8 cm. What is the height, h?



a) Write an equation to represent this problem.

_____

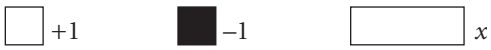
b) Model the equation with balance scales.

c) Solve the equation for h to find the height. _____

7. The area of a rectangle is given by the formula $A = bh$, where b is the base of the rectangle and h is the height. The area of a rectangle is 120 cm^2, and its base is 15 cm. What is the height of the rectangle? _____

Quick Review

As with balance scales, algebra tiles can be used to model and solve equations.

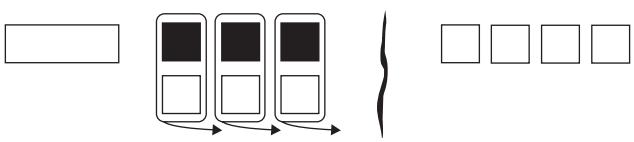


The +1 tile and –1 tile are called **unit tiles**. The x-tile is a **variable tile**.

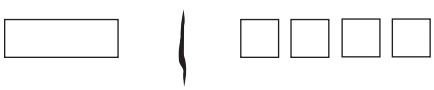
One white unit tile and one black unit tile form a **zero pair**.

To solve the equation $x - 3 = 1$, use tiles to represent the equation.
What you do to one side of the equation, you also do to the other side.

Isolate the x-tile by adding 3 white tiles to each side.
The tiles on the left side make zero pairs. Remove the zero pairs.



This arrangement becomes:



One x-tile equals 4 white tiles. So, $x = 4$

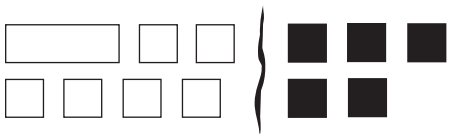
To verify the solution: Replace the variable tile in the original equation with 4 white tiles.

Remove zero pairs. One white tile remains on the left side.
This matches the right side of the equation.
So, the solution is correct.

Practice

1. Match each equation with an arrangement of tiles.

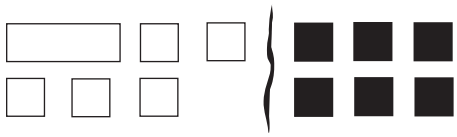
A.



B.

C.

D.



a) $x + 5 = -6$ ____ **b)** $x + 6 = -5$ _____ **c)** $x - 5 = 6$ _____ **d)** $x - 6 = -5$ ____

2. Write the equation modelled by each set of algebra tiles. Then solve the equation.

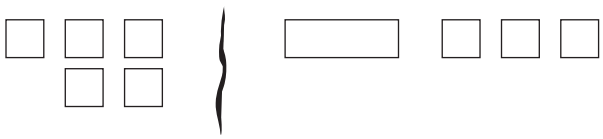
a)

Tip

To isolate the x-tile, make zero pairs.

_____

b)



_____

3. Sketch a set of algebra tiles that represents each equation. Then solve the equation.

a) $x + 3 = 9$ _____ **b)** $3 = x - 5$ _____

4. Use tiles to solve each equation. Verify each solution.

a) $3 + x = 9$
$x =$ __

b) $x - 3 = 9$
$x =$ __

c) $5 + x = 7$
$x =$ __

d) $7 = x - 5$
$x =$ __

5. Solve each equation.
Use tiles to help you.
Verify each solution.

a) $8 = n - 6$
$n =$ ___

b) $n + 5 = 3$
$n =$ ___

c) $7 = n - 8$
$n =$ ___

d) $n + 9 = -4$
$n =$ ___

6. **a)** Eight less than a number is 10.
Let n represent the number.
Then, an equation is: $n - 8 = 10$
Solve the equation.
What is the number? _____

b) Sixteen more than a number is 22.
Write an equation, then solve it to find the number.

_____

7. Between 5 P.M. and midnight, the temperature dropped by 7°C to –5°C.

a) Write an equation you can solve to find the temperature at 5 P.M. _____

b) Use tiles to solve the equation. _____

8. Jamal thinks of an integer.
He adds 8 to this number and the sum is 3.
What is the integer?
Write an equation, then solve it using algebra tiles.

_____

9. Solve each equation. Verify each solution.

a) $x - 25 = 34$

_____

b) $x - 132 = -97$

_____

c) $54 = 130 + x$

_____

d) $176 + x = -24$

_____

6.4 Solving Equations Using Algebra

Quick Review

When you use *algebra* to solve an equation, you always perform the same operation on both sides of the equation. That is, whatever you do to one side of an equation, you must do the same to the other side.

Five more than three times a number is 23.
What is the number?

Let x represent the number.
Then 3 times the number is: $3x$
Five more than 3 times the number is: $3x + 5$
The equation is: $3x + 5 = 23$
Here are the steps to solve this equation:

Step 1: Isolate the variable by adding to or subtracting from each side.
In this case, to remove +5 from the left side, subtract 5 from each side.

$$3x + 5 - 5 = 23 - 5$$
$$3x = 18$$

Step 2: Divide each side by the numerical coefficient.
In this case, divide each side by 3.

$$\frac{3x}{3} = \frac{18}{3}$$

$$x = 6$$

Step 3: Verify the solution by substitution.

Left side $= 3x + 5$ Right side $= 23$
$= 3(6) + 5$
$= 23$

Since the left side equals the right side, $x = 6$ is correct.
The number is 6.

In some equations, such as $5x = 40$, you can omit Step 1 because the variable term is already isolated. In this case, start with Step 2 and divide each side by 5 to get $x = 8$.

Practice

1. Solve each equation.

a) $8x - 7 = 9$

$8x - 7 +$ _____ $= 9 +$ _____

$8x =$ _____

$x =$ _____

b) $9 = 3n - 6$

$n =$ _____

2. Four less than two times a number is 6.
What is the number?

Let x represent the number.
Then 2 times the number is: ___
Four less than 2 times the number is: _____

The equation is: _____

Solve this equation:
To remove _____ from the left side, add _____ to each side.

$2x$ _____ $= 6 +$ _____
$2x \quad = 10$

Divide each side by _____

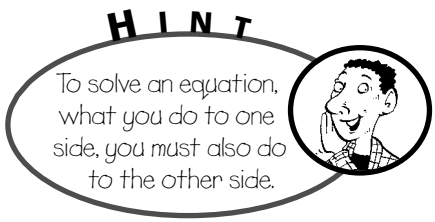
$x =$ ___

Verify the solution.
Left side $= 2x - 4$ Right side $= 6$

$= 2(\_\_) - 4$

$=$ ___

Since the left side equals the right side, $x =$ _____ is correct.
The number is _____.



3. Write, then solve, an equation to answer each question. Verify the solution.

a) Twice a number added to 8 is 14. Let n represent the number.

Equation: _____

To verify the solution, substitute $n =$ _____ into the equation.

Left side: Right side:

b) Fourteen less than four times a number is equal to 6.
Let y represent the number.

Equation: _____

To verify the solution, substitute $y =$ _____ into the equation.
Left side: Right side:

4. Solve each equation. Show your steps. Verify your solution.

a) $3w = 15$

_____

b) $2x = 28$

_____

c) $5y = 40$

_____

d) $8z = 56$

_____

5. Solve each equation. Show your steps. Verify your solution.

a) $3w - 2 = 13$

_____

b) $2x - 4 = 12$

_____

c) $5y - 6 = 14$

_____

d) $7z - 16 = 12$

_____

6. Solve each equation. Show your steps. Verify your solution.

a) $2w + 5 = 11$

_____

b) $3x + 2 = 17$

_____

c) $5y + 6 = 26$

_____

d) $4z + 10 = 30$

_____

7. Solve each equation. Verify your solution.

a) $2w - 5 = 11$ _____

b) $5x + 12 = 52$ _____

c) $13y = 91$ _____

d) $6z - 15 = 57$ _____

8. Write, then solve an equation to find each number. Verify your solution.

a) Seven less than three times a number is 17.

_____

b) Eight more than four times a number is 20.

_____

9. For each problem, write an equation you can use to solve the problem. Solve the equation. Verify the solution.

a) Max has \$34 in his bank account.
He plans to deposit \$12 a week until he has \$130.

How many weeks will it take him? _____

b) Kenji is saving nickels in a jar. He has \$35 in nickels.

How many nickels are in the jar? _____

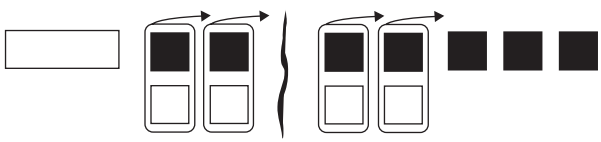
6.5 Using Different Methods to Solve Equations

Quick Review

At Home At School

➤ Use algebra tiles to solve equations that involve integers.
For example, to solve $x - 2 = -5$:

Add 2 white tiles to each side, then remove zero pairs.



The solution is $x = -3$.

➤ Solve *by inspection* for simple equations that involve whole numbers.
For example, to solve $2x + 1 = 9$:
Think: Which number added to 1 gives 9?
Answer: $8 + 1 = 9$
Now think: Which number multiplied by 2 gives 8?
Answer: $4 \times 2 = 8$
So, the solution is $x = 4$.

➤ Use *algebra* to solve any equation.
For example, to solve $\frac{n}{5} = 10$:

$\frac{n}{5} = 10$ Multiply each side of the equation by 5 to isolate n.

$\frac{n}{5} \times 5 = 10 \times 5$

$n = 50$

➤ Use *systematic trial* to solve an equation if you are not sure how to start.

➤ Use the *balance-scales model* to help you visualize the equation.
Whichever method you choose, always *verify* your solution by substituting the solution into the original equation.

Practice

For questions 1 to 4, use algebra, a balance-scales model, inspection, systematic trial, or tiles to solve each equation.
Verify your solutions.

1. **a)** $\frac{x}{3} = 12$

The solution is: _____

b) $\frac{x}{2} = 8$

The solution is: _____

c) $3x = 12$

The solution is: _____

d) $2x = 8$

The solution is: _____

2. **a)** $n - 5 = -3$

The solution is: _____

b) $n + 10 = 6$

The solution is: _____

c) $n + 8 = -2$

The solution is: _____

d) $n - 6 = -10$

The solution is: _____

3. **a)** $2x + 5 = 19$

The solution is: _____

b) $7x + 4 = 18$

The solution is: _____

c) $4x - 3 = 13$

The solution is: _____

d) $3x - 10 = 14$

The solution is: _____

4. **a)** $p + 4 = 11$

The solution is: _____

b) $t - 6 = 14$

The solution is: _____

c) $\frac{k}{8} = 5$

The solution is: _____

d) $5x = 45$

The solution is: _____

For questions 5 to 9, write then solve an equation to solve the problem.

5. One adult ticket costs \$5. One child ticket costs \$3.
The total cost of 2 adult tickets and n child tickets is \$25.
How many child tickets are there?

_____

6. Four years ago, Ellie was 12 years old.
How old is Ellie now?

_____

7. A square has perimeter 28 cm.
What is the length of a side of the square?

_____

8. Phillipe shared his beads with three friends.
Each person had 6 beads.

How many beads did Phillipe start with?

_____

9. Julie sorted 52 sports cards. She divided them into 5 equal groups.
Julie had 12 cards left over. How many cards were in each group?

_____

10. Write a problem that can be described by the equation $2x + 3 = 21$.
Solve the equation. Solve the problem.

_____

_____

_____

In Your Words

Here are some of the important mathematical words of this unit.
Build your own glossary by recording definitions and examples here. The first one is done for you.

solve by inspection

I solve an equation by thinking about the numbers in the equation and how they are related. For example, to solve $x + 7 = 10$; I think, "Which number do I add to 7 to get 10?" The answer, 3, is the solution to the equation, so I write $x = 3$.

solve by systematic trial

verify a solution

balance-scales model

preserving equality

solve using algebra

List other mathematical words you need to know.

Unit Review

LESSON

6.1 **1.** Write an equation you can use to solve each problem.
Solve each equation by inspection or systematic trial.

a) Gabrielle wants to buy a new snowboard that costs $300.
She has $180 in her bank account.
How much more must Gabrielle save so she can buy the snowboard?

_____

b) Freddy bought a new music player for $250.
He then had $380 left in his bank account.
How much was in Freddy's account before he bought the player?

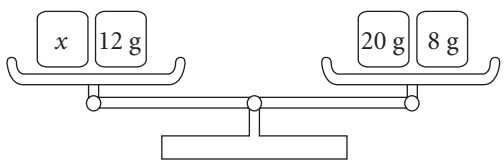
_____

c) Emily helps clean a local yoga studio.
She earns $8 per hour.
Last month Emily got a $10 bonus.
Her total earnings were $170.
How many hours did Emily work?

_____

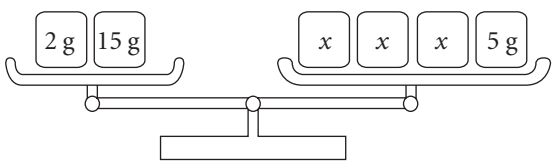
6.2 **2.** Write an equation that is represented by each balance scales.
Solve the equation. Sketch the steps.

a)



_____

b)



_____

LESSON

6.3 **3.** Solve each equation using algebra tiles. Sketch the tiles you used.
Verify each solution.

a) $x + 8 = 5$ **b)** $6 = x - 3$ **c)** $-3 = x + 7$ **d)** $x - 2 = -5$

_____ _____ _____ _____

4. Overnight, the temperature dropped by 15°C to –10°C.

a) Write an equation you can solve to find the temperature before it dropped.

_____

b) Use tiles to solve the equation. _____

6.4 **5.** Solve each equation using algebra. Verify each solution.

a) $4n = 64$ **b)** $2p + 15 = 21$ **c)** $5r - 4 = 26$ **d)** $60 = q + 15$

_____ _____ _____ _____

6. Dylan starts with $40. He saves $12 a week.
After how many weeks will Dylan have each amount?

a) $100 _____ **b)** $136 _____

6.5 **7.** Write an equation for each problem. Solve the equation. Verify each answer.

a) A number increased by 7 is 22. What is the number?

_____

b) William arranges a number of stamps into 5 groups.
There are 12 stamps in each group. How many stamps did William start with?

_____

c) Six less than a number is 25. What is the number?

_____

d) A rectangle has a perimeter of 38 cm. The base is 7 cm.
Sketch and label the rectangle. What is its height?

_____